

Name: Surendra Sharma
 PUL078BCE178
 Theory of Structures-II
 (Pradeep Kojala Sir)

Force method:

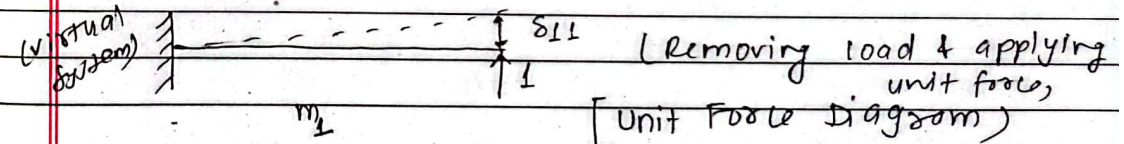
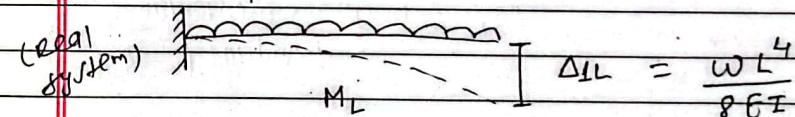
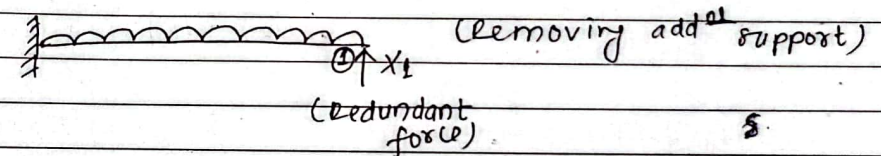
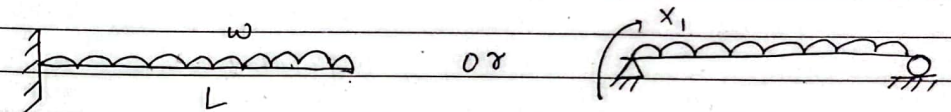


(Propped cantilever)
 [Indeterminate]

How can it be made Determinate?

(Primary structure) / Released structure

Determinate str. formed by removing the cause (may be support or addition of hinge) from given Indeterminate structure



$$\Delta_{IL} = \int \frac{M_L \cdot m_1}{EI} dx$$

δ_{11}
Force position
of displacement

$$\Delta_{1L} = \int_0^L \frac{(-\omega x^2)}{2} (\alpha) dx$$

$$= -\frac{\omega L^4}{8EI}$$

$$\delta_{11} = \int \frac{m_1^2 dx}{EI} \quad (\text{same real \& virtual system to } m_1^2)$$

$$= \frac{L^3}{3EI}$$

= flexibility coefficient.

• Displacement due to unit force $\rightarrow$ flexibility

• Stiffness $\rightarrow$ force for unit displacement

Compatibility Condition

$$\Delta_1 = 0$$

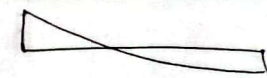
or, $\Delta_{1L} + \Delta_{1R} = 0$ [$\Delta_{1R} \rightarrow$ displacement due to redundant]

$$\text{or, } \frac{-\omega L^4}{8EI} + \delta_{11} \cdot X_1 = 0$$

$$\text{or, } \frac{-\omega L^4}{8EI} + \frac{L^3}{3EI} \cdot X_1 = 0$$

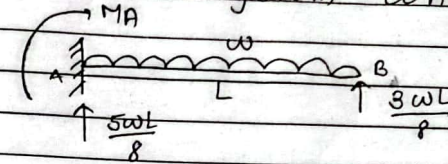
$$\text{or, } \left[X_1 = \frac{3\omega L}{8} \right]$$

X_1 = redundant force to be applied at support.



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Thus, our system converted to



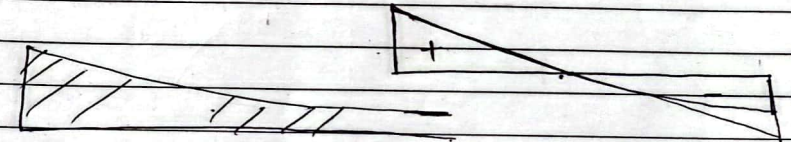
$$\sum M_B = 0 \quad (\rightarrow +ve)$$

$$\text{or, } M_A + \frac{5\omega L}{8} \times L - \frac{\omega L^2}{2} = 0$$

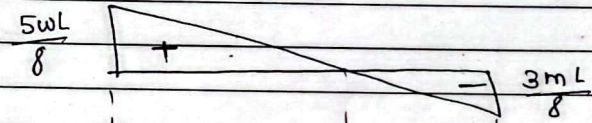
$$\text{or, } \left[M_A = -\frac{\omega L^2}{8} \right]$$

Now, SFD & BMD

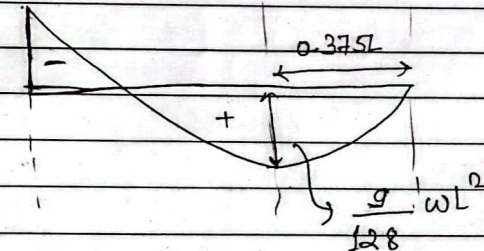
SFD



SFD



BMD



δ_{11} Force position
of displacement

$$\Delta_{11} = \int_0^L \frac{(-wx^2)(x) dx}{EI}$$

$$= -\frac{wL^4}{8EI}$$

$$\delta_{11} = \int \frac{m_1^2 dx}{EI} \quad (\text{same real \& virtual system } \propto m_1^2)$$

$$= \frac{L^3}{3EI}$$

= flexibility coefficient.

• Displacement due to unit force $\rightarrow$ flexibility

• Stiffness $\rightarrow$ force for unit displacement

Compatibility Condition

$$\Delta_1 = 0$$

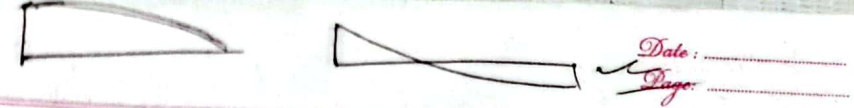
or, $\Delta_{1L} + \Delta_{1R} = 0$ [$\Delta_{1R} \rightarrow$ displacement due to redundant]

$$\text{or, } \frac{-wL^4}{8EI} + \delta_{11} \cdot X_1 = 0$$

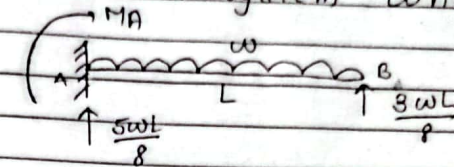
$$\text{or, } \frac{-wL^4}{8EI} + \frac{L^3}{3EI} \cdot X_1 = 0$$

$$\text{or, } \left[X_1 = \frac{3wL}{8} \right]$$

X_1 = redundant force to be applied at support.



Thus, our system converted to



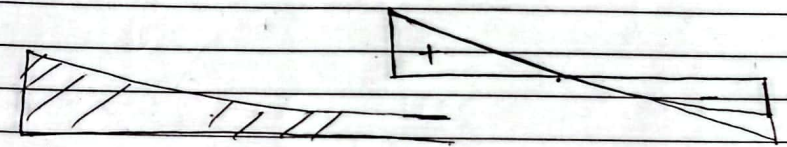
$$\sum M_B = 0 \quad (\uparrow +ve)$$

$$\text{or, } M_A + \frac{5wL}{8} \times L - \frac{wL^2}{2} = 0$$

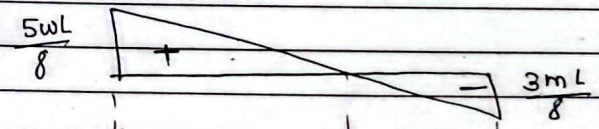
$$\text{or, } \left[M_A = -\frac{wL^2}{8} \right]$$

NOW, SFD & BMD

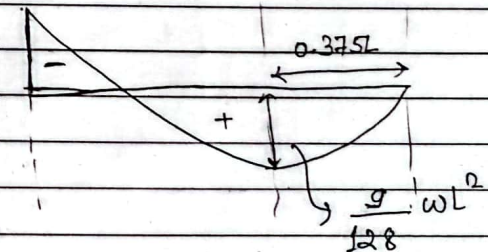
SFD

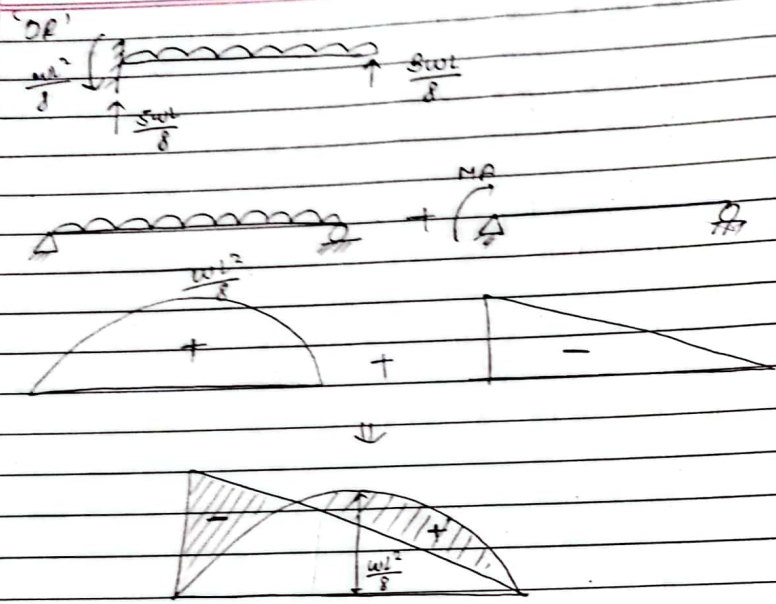


SFD



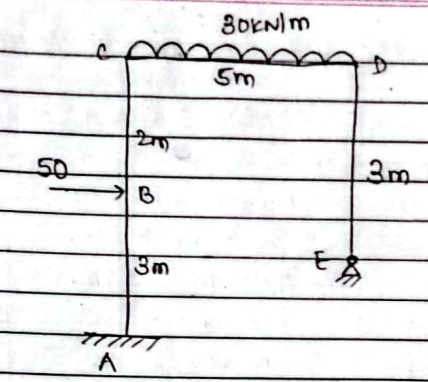
BMD





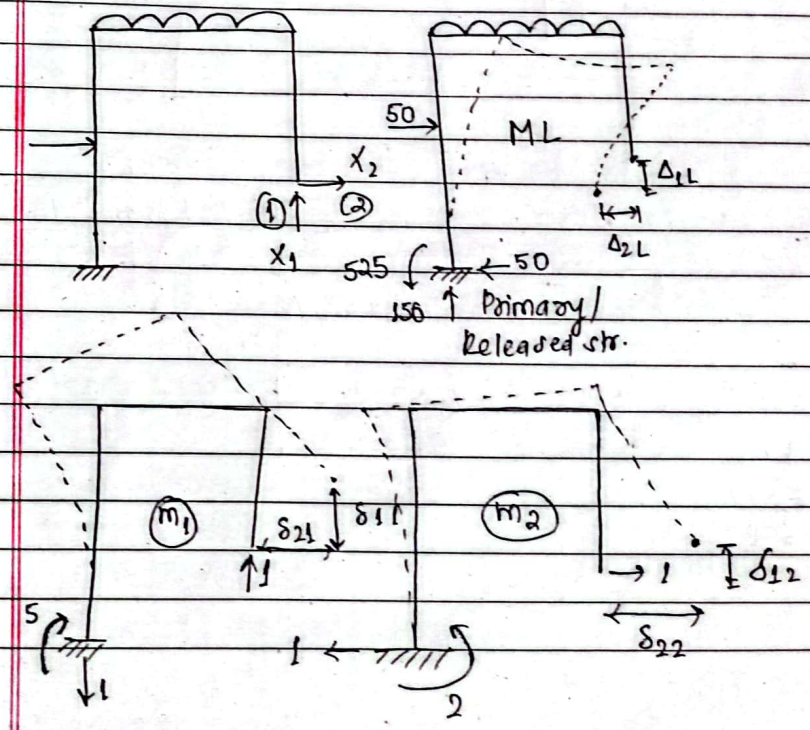
W/w

eg



$$\begin{aligned} \text{DSI} &= \text{NO. of unknowns} - 3 \\ &= 5 - 3 \\ &= 2 \end{aligned}$$

converting to primary str.



2 → H₂
1 → H₁

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According to Maxwell's reciprocal equation,

(1 coordinate at (1) force at (2) displacement =
2 coordinate at (1) force at (2) displacement)

Portion	Origin	Limit	M ₁	m ₁	m ₂
AB	A	0-3	50x-525	5	x-2
BC	B	0-2	150-525 =-375	5	(x+3)-2 =x+1
CD	C	0-5	-375+150x 150x-15x ²	5-x	3
DE	E	0-3	0	0	x

$$\Delta_{21} = \sum \int \frac{M_1 m_2}{EI} dx$$

$$= \int_0^3 \frac{(50x-525)(x-2)}{EI} dx + \int_0^2 \frac{(-375)(x+1)}{EI} dx$$

$$+ \int_0^5 \frac{(-375+150x-15x^2)(3-x)}{EI} dx$$

$$= \frac{1575}{2EI} + \frac{(-1500)}{EI} + \frac{(-625) \times 3}{EI}$$

$$= \frac{-5175}{2EI}$$

NOT / 2EI? → Real system

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$$\Delta_{11} = \sum \int \frac{M_1 m_1}{EI} dx$$

$$= \int_0^3 \frac{(50x-525)(5)}{EI} dx + \int_0^2 \frac{(-375)(5)}{EI} dx$$

$$+ \int_0^5 \frac{(-375+150x-15x^2)(5-x)}{EI} dx$$

$$= \frac{-6750}{EI} + \frac{-3750}{EI} + \frac{-9375}{4EI} = \frac{-25687.5}{2EI}$$

Flexibility Coeff.

$$\delta_{11} = \sum \int \frac{m_1^2}{EI} dx$$

$$= \int_0^3 \frac{5^2}{EI} dx + \int_0^2 \frac{5^2}{EI} dx + \int_0^5 \frac{x^2}{EI} dx$$

$$= \frac{75}{EI} + \frac{50}{EI} + \frac{125}{3EI} = \frac{500}{3EI}$$

$$\delta_{22} = \sum \int \frac{m_2^2}{EI} dx$$

$$= \int_0^3 \frac{(x-2)^2}{EI} dx + \int_0^2 \frac{(x+1)^2}{EI} dx + \int_0^5 \frac{3^2}{EI} dx$$

$$+ \int_0^3 \frac{x^2}{EI} dx = \frac{3}{EI} + \frac{26}{3EI} + \frac{45}{EI} = \frac{197.5}{EI}$$

$$\delta_{12} = \delta_{21} = \sum \int \frac{m_1 m_2}{EI} dx$$

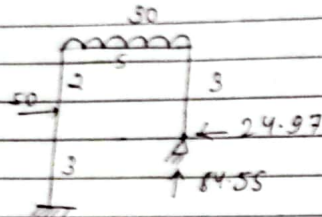
$$= \int_0^3 \frac{(5)(x-2)}{EI} dx + \int_0^2 \frac{(5)(x+1)}{EI} dx$$

$$+ \int_0^5 \frac{(5-x)(3)}{EI} dx$$

$$= \frac{-15}{2EI} + \frac{20}{EI} + \frac{75}{2EI} = \frac{50}{EI}$$

Compatibility equation:

$$\left. \begin{aligned} \Delta_1 &= 0 \\ \Delta_{1L} + \delta_{12}x_2 &= 0 \\ \Delta_{1L} + \delta_{11}x_1 + \delta_{12}x_2 &= 0 \end{aligned} \right\} \begin{aligned} \Delta_2 &= 0 \\ \Delta_{2L} + \delta_{22}x_2 &= 0 \\ \Delta_{2L} + \delta_{21}x_1 + \delta_{22}x_2 &= 0 \end{aligned}$$



From compatibility eqⁿ:

$$\left(\frac{500}{3EI}\right)x_1 + \left(\frac{50}{EI}\right)x_2 = \left(\frac{25687}{2EI}\right) \quad \text{--- (i)}$$

$$\left(\frac{50}{EI}\right)x_1 + \left(\frac{1978}{3EI}\right)x_2 = \left(\frac{5175}{2EI}\right) \quad \text{--- (ii)}$$

Solving,

$$x_1 = 84.55$$

$$x_2 = -24.97$$

* Expressing compatibility equation in matrix form

$$\delta_{11}x_1 + \delta_{12}x_2 + \Delta_{1L} = \Delta_1$$

$$\delta_{21}x_1 + \delta_{22}x_2 + \Delta_{2L} = \Delta_2$$

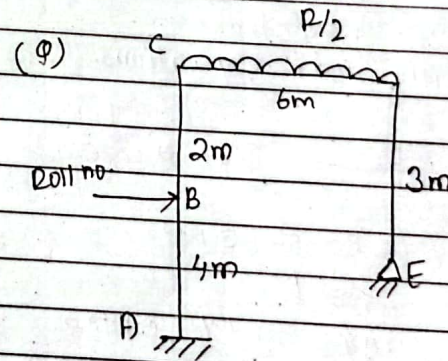
$$\begin{matrix} \text{A} & \text{B} & \text{C} \\ \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} & \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} & = \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} - \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \end{bmatrix} \end{matrix}$$

$\uparrow$ Flexibility matrix $\uparrow$ Force matrix $\uparrow$ net displacement $\uparrow$ Displacement due to load.

$$[S][X] = [\Delta] - [\Delta_L]$$

$$[X] = [S]^{-1} \{ [\Delta] - [\Delta_L] \}$$

$$A \times [B] = [A] \times [C]$$



Draw AFD, SFD, BMD

Primary structure

↳ It is a stable and statically determinate structure obtained by removing the extra supports and connections from the given statically indeterminate structure.

1.8 Indeterminacy and its types

Static Indeterminacy

Number of unknown forces $\rightarrow$ Number of equations of static equilibrium

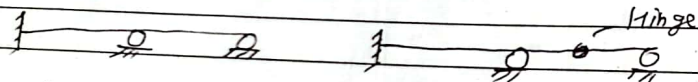
$DSI = \text{Number of extra unknowns}$

Beam:

$$DSI = 8 - 3$$

$8 = \text{Number of reactions}$

$3 = \text{NO. of equilibrium equations}$

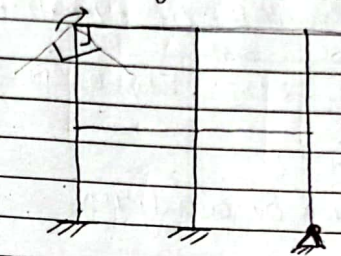


$$DSI = 5 - 3 = 2$$

$$DSI = 5 - (3 + 1) = 1$$

Hinge adds extra equation of condition.

Frame (Rigid Jointed Frame)



$$DSI = DSJE + DSII$$

$DSJE = \text{due to reactions at support \& connections} = 8 - 3$

eg:

$$DSJE = (3 + 3 + 2) - 3 = 5$$

(eg1)

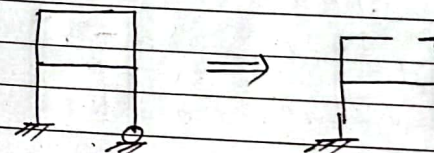
$DSII = \text{NO. of unknowns seen opening each of the closed loop.}$

(converting to open tree system)

(Need Break)

going & returning path same

(the system is determinate)



(open tree) $\rightarrow$ Contiguous system

$$DSII = 3 \times \text{number of closed system}$$

eg:

$$DSII = 3 \times 2 = 6$$

Thus,

$$DSI = DSII + DSJE$$

$$= 6 + 5$$

$$= 11$$

DSI = total number of unknowns
- total number of equation
each joint $\rightarrow 3$ eqs
($\sum F_x = 0, \sum F_y = 0, \sum M = 0$)

$$DSI = (3m + r) - (3j + c)$$

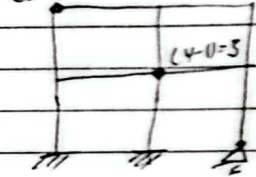
c = equation of condition

eg:

$$DSI = (3 \times 10 + 8) - (3 \times 9)$$

$$= 11$$

If hinge,
 $c = 1$



$$[c = m - 1]$$

Here,

$$DSI = 3 \times 2 - c$$

$\swarrow$ No. of closed loop
 $\nwarrow$ No. of condition equation.

$$= 3 \times 2 - 4$$

$$= 2$$

$$DSIE = 8 - 3 = 5$$

$$DSI = 5 + 2 = 7$$

'or'

$$DSI = (3 \times 10 + 8) - (3 \times 9 + 1)$$

$$= 7$$

If cable



Cable $\rightarrow$ don't consider it as member

due to cable extra

tension force comes.

Thus, DSIE gets decreased by 1

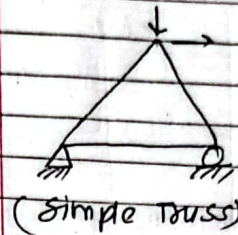
$$DSIE = 8 - 3 + 1$$

$$= 6$$

$$DSI = 6$$

$$DSI = 12$$

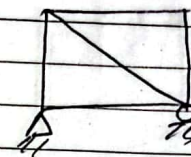
• Truss (Pin jointed Frame)



$$DSI = (m + r) - 2j$$

here,

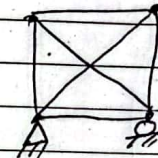
$$DSI = (3 + 3) - 2 \times 3 = 0$$



$$DSI = (m + r) - 2j$$

$$= (5 + 3) - 2 \times 4$$

$$= 0$$



$$DSI = (m + r) - 2j$$

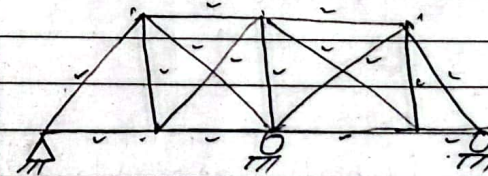
$$= (6 + 3) - 2 \times 4$$

$$= 9$$

$$DSI(I) = m - 2j + 3$$

$$DSI(E) = r - 3$$

eg:



$$m = 15$$

$$j = 8$$

$$r = 4$$

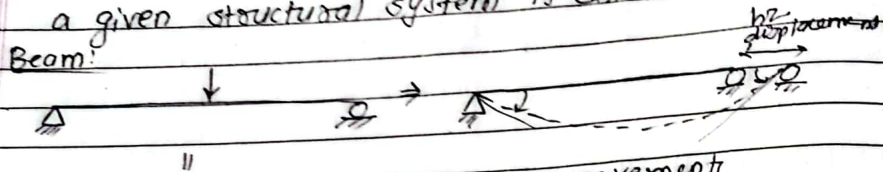
$$DSI(I) = 15 - 2 \times 8 + 3$$

$$= 2$$

$$DSI(E) = r - 3 = 4 - 3 = 1$$

$$DSI = 2 + 1 = 3$$

DKI → Degree of kinematic indeterminacy
 → Number of unknown displacements at joints of a given structural system is called DKI.



3 movements
 (2 rotation
 1 translation)

→ No. of movement possible in a system -
 No. of restraints

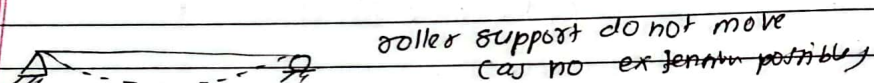
$$DKI = 3j - r = 3 \times 2 - 3 = 3$$

[3 movements possible]

→ If members are inextensible

$$DKI = 3j - (r + m)$$

m → number of members



$$DKI = 3 \times 2 - (3 + 1) = 2$$

eg:

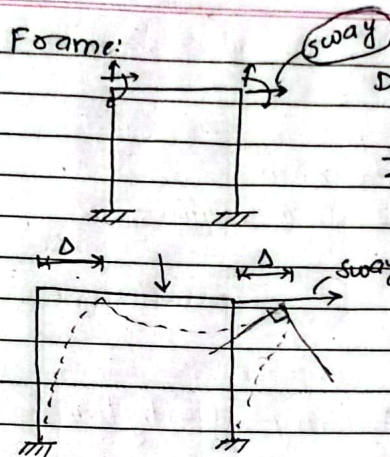
$$DKI = 5 - 3 = 2$$

$$DKI (\text{inextensible}) = 3 \times 3 - (8 + 1) = 2$$

$$DKI (\text{extensible}) = 3 \times 3 - 5 = 4$$

$$\begin{bmatrix} j=3 \\ m=2 \\ r=4 \end{bmatrix}$$

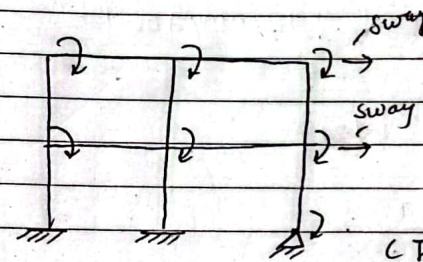
Frame:



$$DKI = 3j - r = 3 \times 4 - 6 = 6$$

$$\text{Inextensible: } DKI = 3j - (r + m) = 3 \times 4 - (6 + 3) = 3$$

eg:



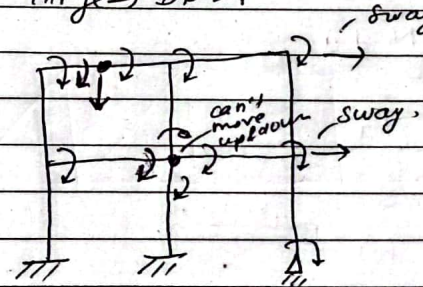
$$\begin{aligned} j &= 9 \\ m &= 10 \\ r &= 8 \end{aligned}$$

$$DKI = 3 \times 9 - (8 + 10) = 9$$

(Inextensible)

eg:

Hinge → DKI ↑



$$DKI = 15$$

(for each hinge 3 · DKI gets increased)

[Hinge → free to move/rotate
 if no support can move up & down]

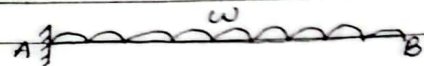
If roller support it might displace ^{Displacement} giving two movements ^{Page} (extra study)

ch. 1
Note

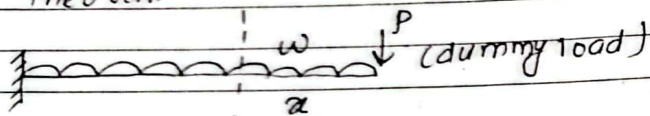
Theorems

- (i) Betti's Theorem
- (ii) Maxwell's reciprocal theorem
- (iii) Castigliano's Theorem I & II. (Numerical)

Q.



Find deflection at B using Castigliano's theorem



$$M_x = -Px - \frac{wx^2}{2}$$

$$U = \int \frac{M^2 dx}{2EI}$$

$$\frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \left(\int \frac{M^2 dx}{2EI} \right)$$

$$= \int \frac{2M}{2EI} \frac{\partial M}{\partial P} dx$$

$$= \int \frac{M}{EI} \frac{\partial M}{\partial P} dx$$

$$\frac{\partial M}{\partial P} = -x \quad \text{for } P=0$$

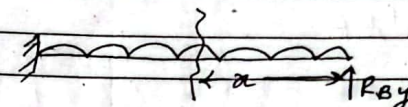
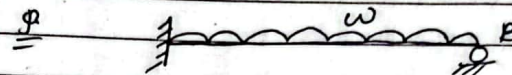
$$\frac{\partial U}{\partial P} = \int_0^L \frac{\left(-\frac{wx^2}{2} \right) (-x)}{EI} dx$$

[When $P=0$]

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$$\frac{\partial U}{\partial P} = \frac{W}{2} \frac{L^4}{4EI}$$

$$\left[\Delta = \frac{\partial U}{\partial P} = \frac{WL^4}{8EI} \right]$$



$$M_x = RBy \cdot x - \frac{wx^2}{2}$$

$$\frac{\partial M_x}{\partial RBy} = x$$

$$\Delta_B = \int \frac{M}{EI} \frac{\partial M}{\partial RBy} dx = 0$$

$$\Delta_B = \int_0^L \frac{\left(RBy \cdot x - \frac{wx^2}{2} \right) (x)}{EI} dx = 0$$

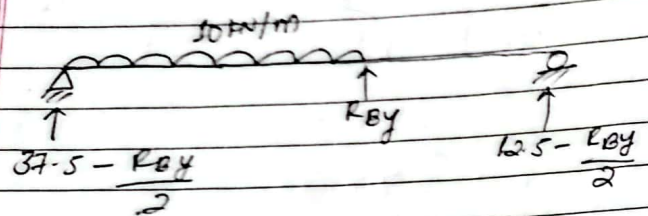
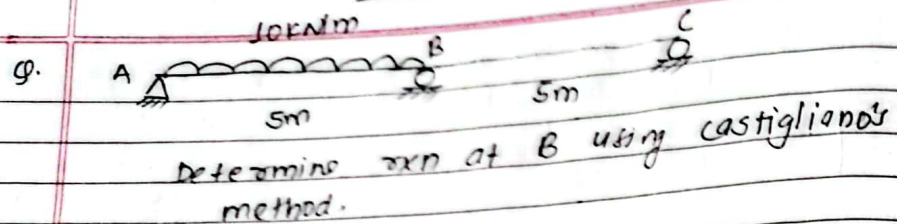
$$\therefore [RBy = \frac{3wL}{8}]$$

In Castigliano's,

Determinate $\rightarrow$ deflection

Indeterminate $\rightarrow$ values of determinates (forces)

075BCE178

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$$\text{AB } M = 37.5x - \frac{R_{By}}{2}x - 5x^2$$

$$\frac{\partial M}{\partial R_{By}} = -\frac{x}{2}$$

$$\text{BC } M = 12.5x - \frac{R_{By}}{2}x$$

$$\frac{\partial M}{\partial R_{By}} = -\frac{x}{2}$$

$$\Delta_B = \sum \int M \frac{\partial M}{\partial R_{By}} \frac{dx}{EI} = 0$$

$$\text{Or, } \textcircled{BC} \int_0^5 \frac{(12.5x - \frac{R_{By}}{2}x)}{EI} x \left(-\frac{x}{2}\right) dx$$

$$\textcircled{AB} + \int_0^5 \frac{(37.5x - \frac{R_{By}}{2}x - 5x^2)}{EI} \left(-\frac{x}{2}\right) dx = 0$$

$$\text{Or, } \frac{1}{EI} \left[\int_0^5 (-6.25x^2 + 0.25x^2 R_{By}) dx + \int_0^5 (-18.75x^2 + 0.25 R_{By} x^2 + 2.5x^3) dx \right] = 0$$

$$\text{Or, } \frac{1}{EI} \left[\left[-\frac{6.25x^3}{3} + \frac{0.25x^3}{3} R_{By} \right]_0^5 + \left[-\frac{18.75x^3}{3} + \frac{0.25x^3}{3} R_{By} + \frac{2.5x^4}{4} \right]_0^5 \right] = 0$$

$$\text{Or, } -\frac{6.25 \times 5^3}{3} + \frac{0.25 \times 5^3}{3} R_{By} - \frac{18.75 \times 5^3}{3} + \frac{0.25 \times 5^3}{3} R_{By} + \frac{2.5 \times 5^4}{4} = 0$$

Solving,

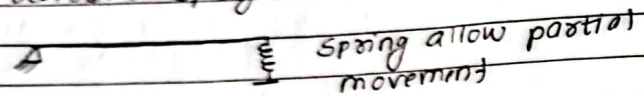
$$[R_{By} = 31.25 \text{ kN}]$$

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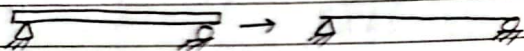
- ↳ Conditions/Requirements of Structural System
→ Strength, Stiffness and Stability (3S)

- ↳ ~~Conditional requirements~~
- ↳ Conditions and equations in structural Analysis
→ Static, compatibility & Physical.

- ↳ Partial restraint Spring



- ↳ Structure idealization



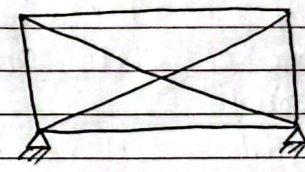
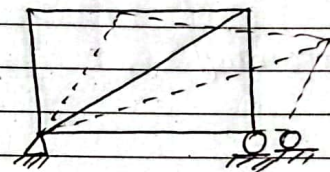
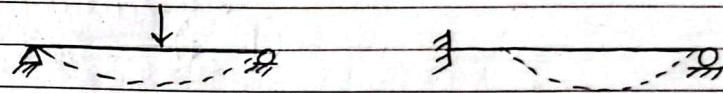
Breadth and height ignored.

- Making some modifications in structure to make subsequent calculations easier
- Global co-ordinate system, single origin point for entire structure
- Local co-ordinate system: Individual origin points for each member of structure.

Indeterminacy of structural system, its physical meaning and types

- ↳ Solvable with static equilibrium eqns → Determinate
- ↳ If a structural system can be analyzed by using equations of static eqn, then the system is said to be statically determinate

- ↳ Physical meaning: Indeterminate structures are more rigid than determinate structures. Indeterminate structures are capable of bearing of more load and more restrained.



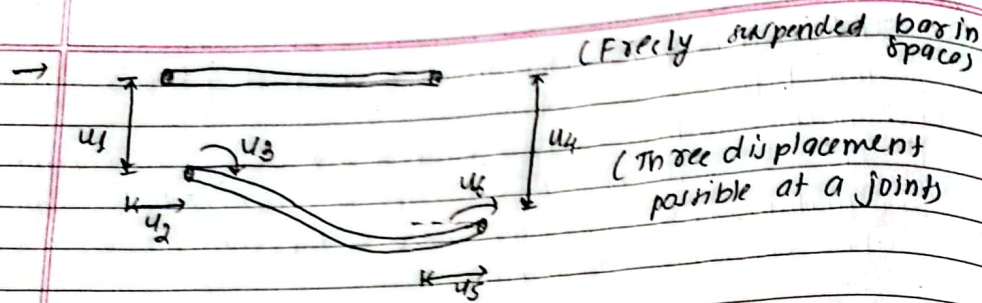
Statically Determinate

Statically Indeterminate

Types

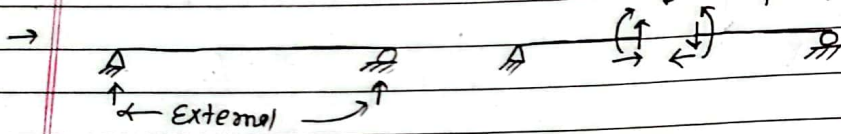
- (1) Static Indeterminacy → Reaction and forces are variable
↳ $DSI = \text{No. of unknown} - \text{No. of known (forces)}$

- (2) Kinematic Indeterminacy → Displacement of joints are variable (slope & deflection/displacement)
↳ $DKI = \text{No. of unknown displacement at joint} - \text{No. of known equations}$



→ only 3 displacements possible in total
So, $DSI = 3j - r$

$r \rightarrow$ reactions
Internal forces

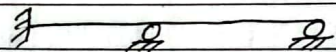


n 1.8 (A) Degree of static Indeterminacy

(a) External

(b) Internal

→ Beam



In beam, only external indeterminacy exists

→ External DSI: indeterminacy due to external forces (Rxn at supports)

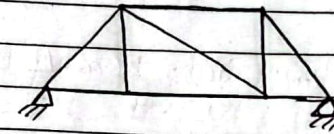
Internal DSI: indeterminacy due to internal forces (internal rxn in member $\rightarrow$ SF, BM, AF)

$$DSI = r - (3 + c)$$

(Unknown-known)

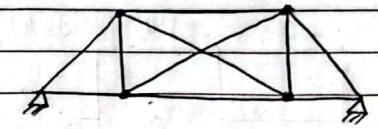
$c \rightarrow$ condition due to internal hinge

* Truss



$$DSI = (m + r) - 2j$$

$2j$ known for truss because in each joint, two eqs are available
 $\sum F_x = 0$ & $\sum F_y = 0$
pin joint $\rightarrow$ can't resist moment



Member could overlap internal indeterminacy

$$(DSI)_{ext} = r - 3 = 4 - 3 = 1$$

(4 unknown - 3 eqs)

$$(DSI)_{int} = (DSI)_{total} - (DSI)_{ext}$$

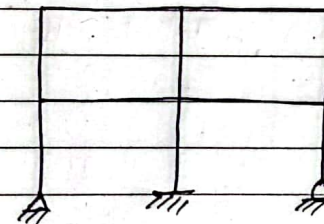
$$(DSI)_{total} = m + r - 2j$$

$$= (10 + 4) - 2 \times 4$$

$$= 2$$

$$(DSI)_{int} = 2 - 1 = 1$$

* Frame



eg. 1

$$DSI = (3m + r) - (3j + \sum (m' - 1))$$

$3j$ because 3 eqs are known
 $\sum F_x = 0, \sum F_y = 0, \sum M = 0$

$$(DSI)_{ext} = r - 3$$

$$(DSI)_{int} = 3 \times c' - \sum (m' - 1)$$

$c' \rightarrow$ No. of closed loop

$m' \rightarrow$ member connected to

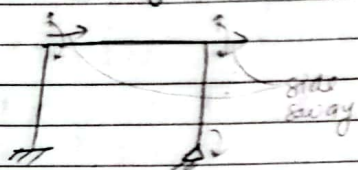
internal hinge.
 $c \rightarrow$ Eq of condition



If extensible, $DKI = 3 \times 3 - 5 = 4$
 If inextensible, $DKI = 3 \times 3 - (5+2) = 2$

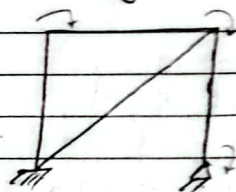
Frame

In exam inextensible must be considered if not given



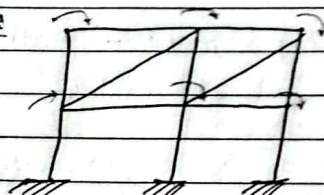
Extensible, $DKI = 3 \times 4 - 5 = 7$
 Inextensible, $DKI = 3 \times 4 - (5+3) = 4$

If bracing is done

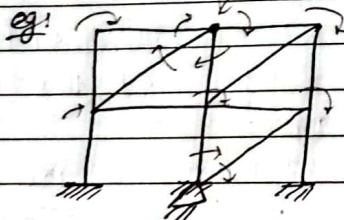


Inextensible,
 $DKI = 3 \times 4 - (5+10) - \text{bracing}$
 $= 12 - (5+3) - 1$
 $= 3$

eg:

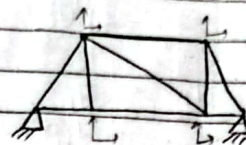


$DKI = 6$
 $= 3 \times 9 - (9+10) - 2$
 $= 6$

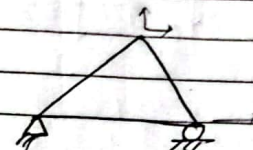


$DKI = 11$
 $= 3 \times 9 - (8+10) - 3$
 (Hinge then can't use formula)

Touss: Member rotation $\frac{Y_i}{L}$



$DKI = 2j - 8$
 $= 2 \times 6 - 4 = 8$



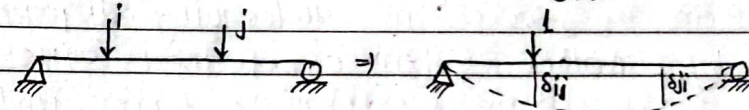
$DKI = 2 \times 3 - 3 = 3$

#

Betti's law and Maxwell's reciprocal theorem

(A) Maxwell's Reciprocal Theorem (MRT)

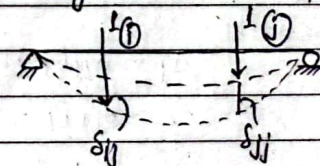
In a linearly elastic structure, displacement at coordinate (i) due to unit force at coordinate (j) i.e. δ_{ij} is equal to displacement at coordinate j due to unit force at coordinate i (δ_{ji})



Proof: Let us apply unit force at coordinate i gradually
 work done, $W_1 = \frac{1}{2} \times 1 \times \delta_{ii} = \frac{\delta_{ii}}{2}$

Again, apply unit force at j gradually while unit force at i is already present

$W_2 = 1 \cdot \delta_{ij} + \frac{1}{2} \times 1 \times \delta_{jj}$



Total work done: $WT = W_1 + W_2 = \frac{\delta_{ii} + \delta_{jj}}{2} + \delta_{ij}$

Again let us apply unit load at J gradually

$$W_1' = \frac{1}{2} \times 1 \times \delta_{ij} = \frac{\delta_{ij}}{2}$$

$$W_2' = \delta_{ji} + \frac{\delta_{ij}}{2}$$

$$W_T' = \delta_{ji} + \frac{\delta_{ij} + \delta_{ji}}{2}$$

For linearly elastic structures

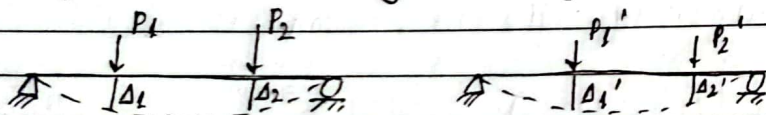
$$W_T = W_T'$$

$$\text{or } \delta_{ji} + \delta_{ji} + 2\delta_{ij} = 2\delta_{ji} + \delta_{ij} + \delta_{ji}$$

$$\text{or } \boxed{\delta_{ij} = \delta_{ji}}$$

(B) Betti's Law

Theorem: In a linearly elastic structure, virtual workdone by system of forces in undergoing displacements caused by another system of forces is equal to virtual workdone by second system of forces undergoing displacements caused by first system of forces.



$$\Delta_1 = \delta_{11}P_1 + \delta_{12}P_2, \quad \Delta_2 = \delta_{21}P_1 + \delta_{22}P_2$$

$$\Delta_1' = \delta_{11}P_1' + \delta_{12}P_2', \quad \Delta_2' = \delta_{21}P_1' + \delta_{22}P_2'$$

Virtual workdone by 1st system of forces while undergoing displacement caused by 2nd system of forces

$$\Sigma P\Delta' = P_1\Delta_1' + P_2\Delta_2'$$

$$= P_1(\delta_{11}P_1' + \delta_{12}P_2') + P_2(\delta_{21}P_1' + \delta_{22}P_2')$$

$$= P_1P_1'\delta_{11} + P_1P_2'\delta_{12} + P_1'P_2\delta_{21} + P_2P_2'\delta_{22}$$

Again,

virtual workdone by 2nd system of forces while undergoing displacement caused by 1st system of forces

$$\Sigma P'\Delta = P_1'\Delta_1 + P_2'\Delta_2$$

$$= P_1'(\delta_{11}P_1 + \delta_{12}P_2) + P_2'(\delta_{21}P_1 + \delta_{22}P_2)$$

$$= P_1'P_1\delta_{11} + P_1'P_2\delta_{12} + P_1P_2'\delta_{21} + P_2'P_2\delta_{22}$$

Using Maxwell Reciprocal Theorem,

$$\delta_{12} = \delta_{21}$$

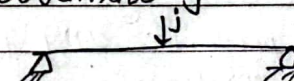
we get

$$[\Sigma P\Delta' = \Sigma P'\Delta]$$

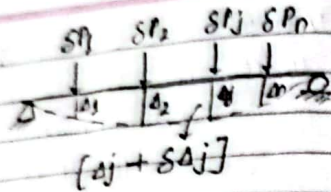
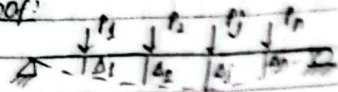
Castigliano's Theorem

Theorem I: The partial derivative of strain energy of linearly elastic structure expressed in terms of displacement with respect to any displacement Δ_j is equal to force at coordinate j.

$$\frac{\partial u}{\partial \Delta_j} = P_j$$



Proof:



Consider several forces $P_1, P_2, \dots, P_j, \dots, P_n$ are acting on structure producing displacements $\Delta_1, \Delta_2, \dots, \Delta_j, \dots, \Delta_n$. This is assumed as 1st system for Betti's theorem. Now, impose small increments $\delta\Delta_j$ to displacement at coordinate j keeping all other displacements unchanged, this is assumed as 2nd system for Betti's theorem.

Applying Betti's Theorem

$$P_1 \cdot 0 + P_2 \cdot 0 + P_j \cdot \delta\Delta_j + P_n \cdot 0 = \delta P_1 \Delta_1 + \delta P_2 \Delta_2 + \delta P_j \Delta_j + \delta P_n \Delta_n$$

$$\Rightarrow P_j \delta\Delta_j = \delta U$$

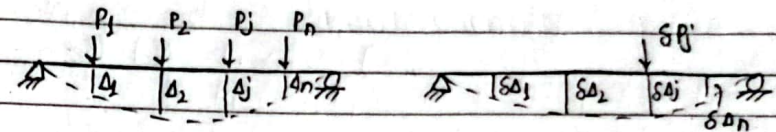
$$\Rightarrow \frac{\delta U}{\delta\Delta_j} = P_j$$

$$\text{or, } \left[\lim_{\delta\Delta_j \rightarrow 0} \frac{\delta U}{\delta\Delta_j} = \frac{\partial U}{\partial \Delta_j} = P_j \right]$$

Theorem II: The partial derivative of strain energy of linearly elastic system expressed in terms of forces with respect to any force P_j is equal to displacement Δ_j at coordinate j .

$$\text{Mathematically } \frac{\partial U}{\partial P_j} = \Delta_j$$

Proof: Consider several forces $P_1, P_2, \dots, P_j, \dots, P_n$ are acting on structure at coordinate $1, 2, \dots, j, \dots, n$ producing displacement $\Delta_1, \Delta_2, \dots, \Delta_j, \dots, \Delta_n$. This is assumed as first system. Now, impose small increment δP_j to force P_j , keeping all other forces unchanged. As a result, increments in displacements are $\delta\Delta_1, \delta\Delta_2, \delta\Delta_j, \delta\Delta_n$. This is assumed to be as 2nd system.



Applying Betti's law,

$$P_1 \delta\Delta_1 + P_2 \delta\Delta_2 + \dots + P_j \delta\Delta_j + \dots + P_n \delta\Delta_n = 0 \times \Delta_1 + 0 \times \Delta_2 + \dots + \delta P_j \times \Delta_j + 0$$

$$\Rightarrow \delta U = \delta P_j \Delta_j$$

$$\Rightarrow \left[\frac{\delta U}{\delta P_j} = \Delta_j \right] \text{ or } \left[\lim_{\delta P_j \rightarrow 0} \frac{\delta U}{\delta P_j} = \frac{\partial U}{\partial P_j} = \Delta_j \right]$$

Uses: (1) TO find displacement (slope, deflections) in statically determinate structure

(2) TO find redundant force in statically indeterminate structure

Beam (Flexure Member): Strain Energy (U) = $\sum \int \frac{M^2 dx}{2EI}$

$$\text{or, } \frac{\partial U}{\partial P} = \sum \int \frac{2M}{2EI} \times \frac{\partial M}{\partial P} \times dx$$

$$\left[\Delta = \sum \int \frac{M}{EI} \frac{\partial M}{\partial P} dx \right]$$

Torsion (Axial Member)

$$U = \frac{\sum P^2 L}{2AE}$$

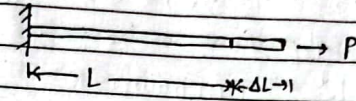
$$\frac{\partial U}{\partial P'} = \frac{\sum 2PL}{2AE} \times \frac{\partial P}{\partial P'}$$

$$\Rightarrow [\Delta = \sum P' \left(\frac{\partial P}{\partial P'} \right) \frac{L}{AE}]$$

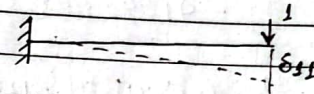
$P' \rightarrow$ dummy load

Flexibility & Stiffness

The displacement at any co-ordinate due to unit force at that co-ordinate is called flexibility (δ)

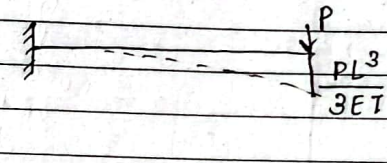


$$\Delta L = \frac{PL}{AE} \Rightarrow \delta = \frac{L}{AE}$$



The force required to produce unit displacement is called stiffness (K).

$$\Delta L = \frac{PL}{AE} \Rightarrow 1 = \frac{KL}{AE} \text{ or } K = \frac{AE}{L} \text{ (stiffness axial)}$$



If $P=1$, $\delta = \frac{L^3}{3EI}$
if $\delta=1$, $K = \frac{3EI}{L^3}$

Flexibility matrix: $\rightarrow$ $\delta_{ij} = 0$ $\delta_{ij} = 0$

$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \Delta_{1L} = \Delta_1$$

$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \Delta_{2L} = \Delta_2$$

$$\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \Delta_{3L} = \Delta_3$$

These equation can be expressed as

$$\begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \\ \Delta_{3L} \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix}$$

Flexibility matrix $[\delta] [X] = [\Delta]$ $\therefore$ This is flexibility matrix

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Page: _____

both flexibility & stiffness matrices are called system specific matrices $\rightarrow$ related to system only, not related to load

Relation between flexibility & stiffness Matrix

$$[\Delta] = [\delta][P]$$

Multiplying by $[\delta]^{-1}$

$$[\delta]^{-1}[\Delta] = [\delta]^{-1}[\delta][P]$$

$$\text{or, } [\delta]^{-1}[\Delta] = [P] \quad \text{--- (i)}$$

Also,

$$[P] = [K][\Delta] \quad \text{--- (ii)}$$

from (i) & (ii),

$$[\delta]^{-1} = [K]$$

Multiplying by $[\delta]$

$$[\delta][\delta]^{-1} = [\delta][K]$$

$$\text{or, } [I] = [\delta][K] \quad \star$$

Stiffness matrix



Unknown, $\Delta_1, \Delta_2, \Delta_3$

For Δ_1 previously it has c/w moment so, rotate member incl w dist. (or at moment at joint) For each rotation joint

$[K]$ Stiffness Force req. to make unit displacement

* Egb^m eqⁿ at coordinate 1, 2 & 3 are

$$\left. \begin{aligned} P_1 + K_{11}\Delta_1 + K_{12}\Delta_2 + K_{13}\Delta_3 &= P_1 \\ P_2 + K_{21}\Delta_1 + K_{22}\Delta_2 + K_{23}\Delta_3 &= P_2 \\ P_3 + K_{31}\Delta_1 + K_{32}\Delta_2 + K_{33}\Delta_3 &= P_3 \end{aligned} \right\} \text{equilibrium equations.}$$

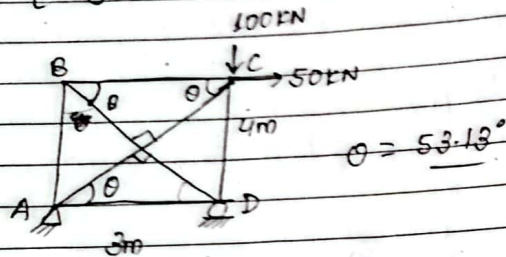
$$\rightarrow \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} + \begin{bmatrix} P_{1L} \\ P_{2L} \\ P_{3L} \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Stiffness matrix $\uparrow$ support at restrained joint

Castigliano's Method for Indeterminate Truss

$$U = \sum \frac{P^2 L}{2AE}$$

$$\left[\frac{\partial U}{\partial P_j} = \sum \frac{P}{AE} \frac{\partial P}{\partial P_j} L = 0 \right]$$



Remove any one redundant
Assume force as X_1 .

Joint Method:

At A:

$$P_{AB} + X_1 \sin \theta = \frac{200}{3} \quad \text{--- (i)}$$

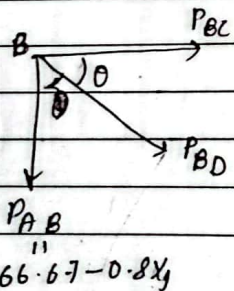
$$P_{AB} = 66.67 - 0.8X_1 \quad \text{--- (ii)}$$

~~P_{AB}~~

$$X_1 \cos \theta + P_{AD} = 50 \quad \text{--- (iii)}$$

$$P_{AD} = 50 - 0.6X_1$$

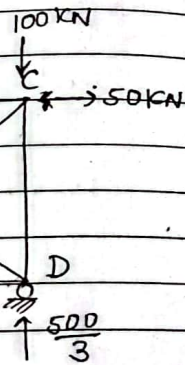
At B:



~~P_{AB}~~

$$66.67 - 0.8X_1 + P_{BD} \sin \theta = 0$$

$$P_{BD} = X_1 - 83.34$$



$$\sum F_x = 0$$

$$P_{BC} + P_{BD} \cos \theta = 0$$

$$\text{or, } P_{BC} + 0.6 P_{BD} = 0$$

$$\text{or, } P_{BC} + 0.6X_1 - 50 = 0$$

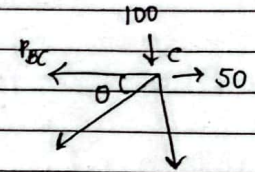
$$P_{BC} = 50 - 0.6X_1$$

At C:

$$\sum F_y = 0$$

$$\text{or, } 100 + P_{CD} + P_{AC} \sin \theta = 0$$

$$\text{or, } P_{CD} = -100 - 0.8X_1$$



$AE = \text{constant}$

Members	L	P	$\frac{\partial P}{\partial X_1}$	$\frac{P}{AE} \left(\frac{\partial P}{\partial X_1} \right) L$
AB	4m	$66.67 - 0.8X_1$	-0.8	$(66.67 - 0.8X_1)(-0.8)(4)$
AD	3m	$50 - 0.6X_1$	-0.6	$(50 - 0.6X_1)(-0.6)(3)$
AC	5m	X_1	1	$(X_1)(1)(5)$
BD	5m	$X_1 - 83.34$	1	$(X_1 - 83.34)(1)(5)$
BC	3m	$50 - 0.6X_1$	-0.6	$(50 - 0.6X_1)(-0.6)(3)$
CD	4m	$-100 - 0.8X_1$	-0.8	$(-100 - 0.8X_1)(-0.8)(4)$

$$\sum \frac{P}{AE} \left(\frac{\partial P}{\partial X_1} \right) L = 0$$

$$\text{or, } -213.344 + 2.56X_1 - 90 + 1.08X_1 + 320 + 2.56X_1$$

$$-90 + 1.08X_1 + 5X_1 - 416.7 + 5X_1 = 0$$

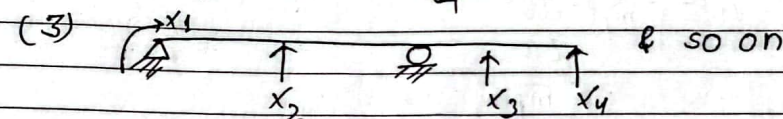
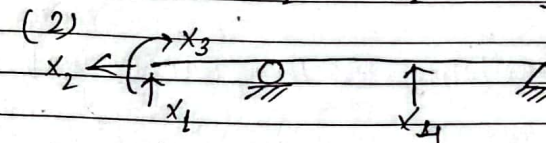
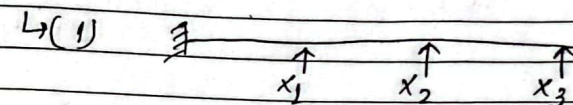
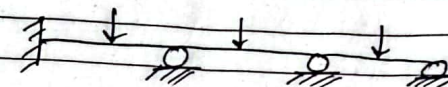
$$\text{Solving, } [X_1 = 28.359 \text{ kN}]$$

Member	P
AB	43.9828
BC	32.9846
CD	-122.6872
DA	32.9846
BD	-54.98
AC	28.559

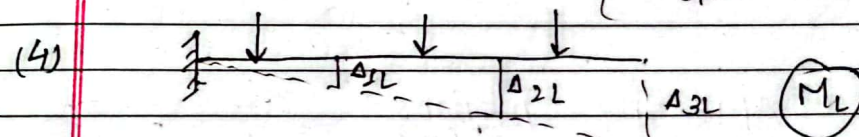
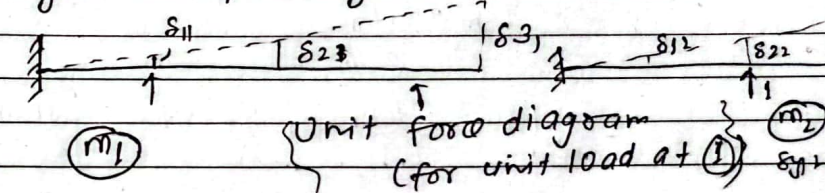
2. Force Method

(1) DSI

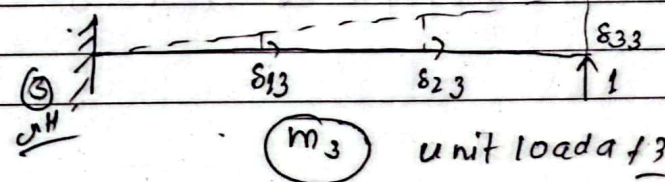
(2) Choose Redundants and make Primary structure



(3) Draw unit force diagram for each co-ordinate. Taking (1) as primary structures.



(for given loading system)



⑤ Calculate displacements due to load at coordinates assigned

$$\Delta_{11} = \int \frac{M_1 m_1 dx}{EI}$$

⑥ Determine flexibility matrix/coefficient.

$$\delta_{11} = \int \frac{m_1^2 dx}{EI}$$

$$\delta_{12} = \delta_{21} = \int \frac{m_1 m_2 dx}{EI}$$

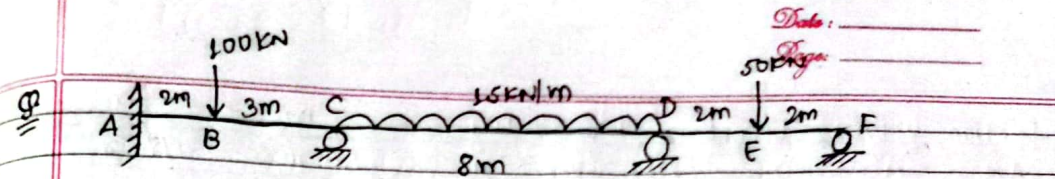
$$\delta_{13} = \delta_{31} = \int \frac{m_1 m_3 dx}{EI}$$

$$\delta_{22} = \int \frac{m_2^2 dx}{EI}$$

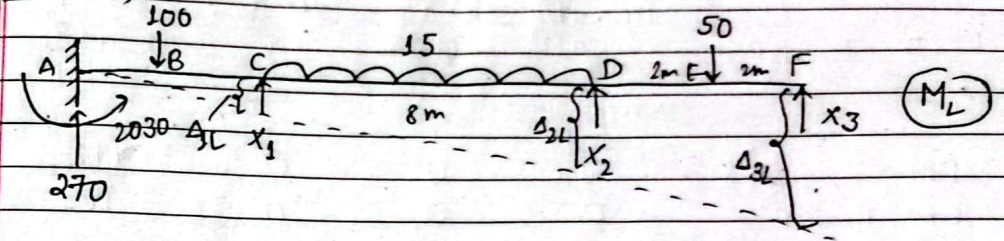
$$\delta_{33} = \int \frac{m_3^2 dx}{EI} ; \delta_{23} = \delta_{32} = \int \frac{m_2 m_3 dx}{EI}$$

Solve Compatibility equations

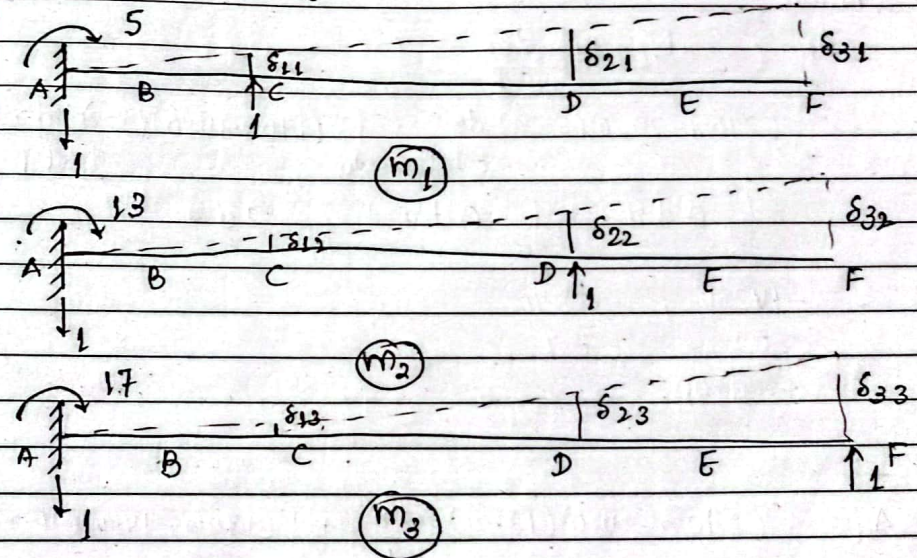
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix}^{-1} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} - \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \\ \Delta_{3L} \end{bmatrix}$$



Solution



Unit force diagram



Portion	Origin	Limit	M_L	m_1	m_2	m_3
AB	A	0-2	$270x-2030$	$5-x$	$13-x$	$17-x$
BC	B	0-3	$270(x+2)$ $-2030-100x$ $=170x-1490$	$5-(x+2)$	$13-(x+2)$	$17-(x+2)$
DECD	D	0-8	$-50(x+2)$ $=-7.5x^2$ $-7.5x^2-50x-100$	0	x	$x+4$
DE	E	0-2	$-50x$	0	0	$x+2$
EF	F	0-2	0	0	0	x

Now,

$$\Delta_{LL} = \sum M_L \cdot m_j \cdot \frac{dx}{EI}$$

$$= \int_0^2 \frac{(270x-2030)(5-x)}{EI} dx + \int_0^3 \frac{(170x-1490)(3-x)}{EI} dx$$

$$+ \int_0^8 \frac{0 \cdot x}{EI} dx + \int_0^2 \frac{0 \cdot x}{EI} dx + \int_0^2 \frac{0 \cdot x}{EI} dx$$

$$= \frac{-14260}{EI} + \frac{-5940}{EI}$$

$$= \frac{-20200}{EI}$$

$$\Delta_{LL} = \int_0^2 \frac{(270x-2030)(13-x)}{EI} dx + \int_0^3 \frac{(170x-1490)(11-x)}{EI} dx$$

$$+ \int_0^8 \frac{(-7.5x^2-50x-100)(x)}{EI} dx$$

$$= \frac{-42420}{EI} - \frac{35580}{EI} - \frac{288540}{3EI} - \frac{58240}{3EI}$$

$$= \frac{-292240}{3EI}$$

$$\Delta_{LL} = \int_0^2 \frac{(270x-2030)(17-x)}{EI} dx + \int_0^3 \frac{(170x-1490)(15-x)}{EI} dx$$

$$+ \int_0^8 \frac{(-7.5x^2-50x-100)(x+4)}{EI} dx + \int_0^2 \frac{(-50x)(x+2)}{EI} dx$$

$$= \frac{-56500}{EI} - \frac{50400}{EI} - \frac{102400}{3EI} - \frac{1000}{3EI}$$

$$= \frac{-424100}{3EI}$$

$$\delta_{11} = \sum \int \frac{m_1^2 dx}{EI} = \int_0^2 \frac{(5-x)^2 dx}{EI} + \int_0^3 \frac{(3-x)^2 dx}{EI} = \frac{125}{3EI}$$

$$\delta_{22} = \sum \int \frac{m_2^2 dx}{EI} = \int_0^2 \frac{(13-x)^2 dx}{EI} + \int_0^3 \frac{(11-x)^2 dx}{EI} + \int_0^8 \frac{x^2 dx}{EI}$$

$$= \frac{2197}{3EI}$$

$$\delta_{33} = \sum \int \frac{m_3^2 dx}{EI} = \int_0^2 \frac{(17-x)^2 dx}{EI} + \int_0^3 \frac{(15-x)^2 dx}{EI} + \int_0^8 \frac{(x+4)^2 dx}{EI}$$

$$+ \int_0^2 \frac{(x+2)^2 dx}{EI} + \int_0^2 \frac{x^2 dx}{EI}$$

$$= \frac{1538}{3EI} + \frac{549}{3EI} + \frac{1664}{3EI} + \frac{56}{3EI} + \frac{8}{3EI} = \frac{4913}{3EI}$$

$$\delta_{13} = \delta_{31} = \sum \int \frac{m_1 \cdot m_3 dx}{EI} = \int_0^2 \frac{(5-x)(17-x)}{EI} dx + \int_0^3 \frac{(3-x)(15-x)}{EI} dx$$

$$= \frac{38}{3EI} + \frac{63}{EI} - \frac{575}{3EI}$$

$$\delta_{32} = \delta_{23} = \int_0^8 m_2 \cdot m_3 \frac{dx}{EI}$$

$$= \int_0^2 (13-x)(17-x) \frac{dx}{EI} + \int_0^3 (11-x)(15-x) \frac{dx}{EI} + \int_0^8 x(x+4) \frac{dx}{EI}$$

$$= \frac{1154}{3EI} + \frac{387}{EI} + \frac{896}{3EI}$$

$$= \frac{3211}{3EI}$$

$$\delta_{13} = \delta_{31} = \int_0^2 m_1 \cdot m_3 \frac{dx}{EI}$$

$$= \int_0^2 (5-x)(17-x) \frac{dx}{EI} + \int_0^3 (3-x)(15-x) \frac{dx}{EI}$$

$$= \frac{386}{3EI} + \frac{63}{EI}$$

$$= \frac{575}{3EI}$$

$$\delta_{12} = \delta_{21} = \int_0^2 m_1 \cdot m_2 \frac{dx}{EI}$$

$$= \int_0^2 (5-x)(13-x) \frac{dx}{EI} + \int_0^3 (3-x)(11-x) \frac{dx}{EI}$$

$$= \frac{425}{3EI}$$

We know,

$$[S][X] + [\Delta_L] = [\Delta]$$

$$\text{or } [X] = \{ [\Delta] - [\Delta_L] \} [S]^{-1}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} - \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \\ \Delta_{3L} \end{bmatrix} \right\}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 125/3 & 425/3 & 575/3 \\ 425/3 & 2187/3 & 3211/3 \\ 575/3 & 3211/3 & 4913/3 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -20200/3 \\ -292240/3 \\ -424100/3 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 0.0906 & -0.0453 & 0.01901 \\ -0.0453 & 0.0531 & -0.0294 \\ 0.01901 & -0.0294 & 0.01762 \end{bmatrix} \begin{bmatrix} 20200 \\ 292240/3 \\ 424100/3 \end{bmatrix}$$

$$= \begin{bmatrix} 103.733 \\ 101.207 \\ 8.035 \end{bmatrix}$$

Thus,

$$X_1 = 103.733 \text{ kN}$$

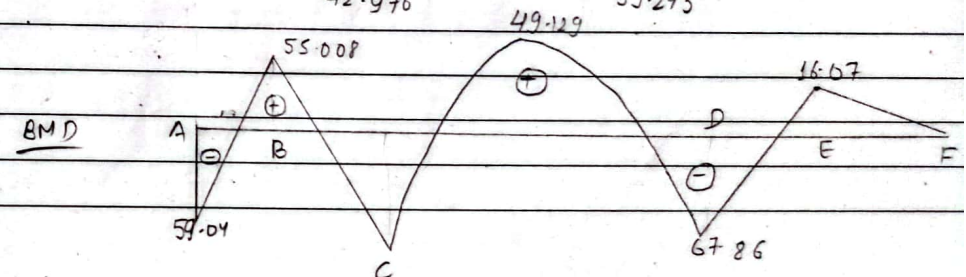
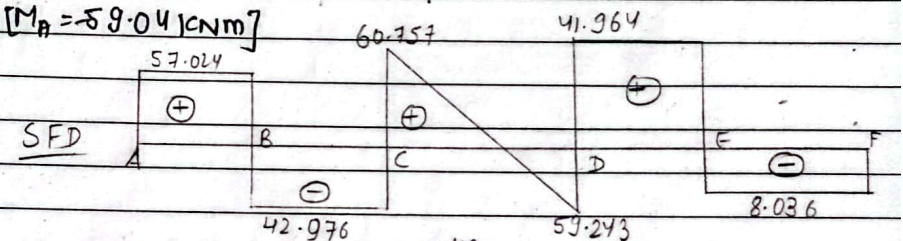
$$X_2 = 101.207 \text{ kN}$$

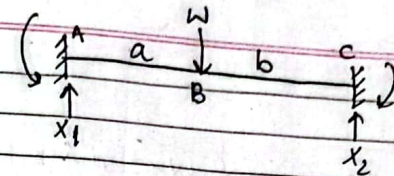
$$X_3 = 8.035 \text{ kN}$$

$$\sum F_y = 0 \text{ gives } R_A = 57.024 \text{ kN}$$

$$\sum M_A = 0$$

$$\Rightarrow [M_A = 59.04 \text{ kNm}]$$





$DSI = 26 - 3 = 3$

$DSI = 2$

NO horizontal force. So,

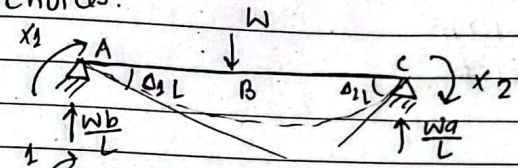
$DSI = 3 - 2 = 1$

$\sum F_y = 0$ & $\sum M = 0$
not horizontal axis

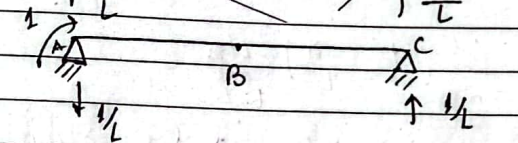
$2 \times 1 = 2$
 $= 4 - 2 = 2$

Choices:

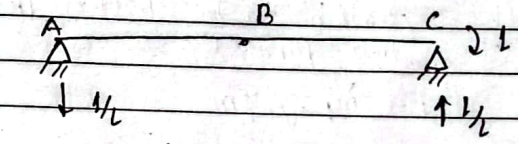
(M_L)



(m₁)



(m₂)



Portion	orig	Limit	M _L	m ₁	m ₂	m ₂ Limit
AB	A	0-a	$\frac{Wb}{L}a$	$1 - \frac{a}{L}$	$-\frac{a}{L}$	0-a
BC	C	0-b	$\frac{Wa}{L}b$	$\frac{b}{L}$	$\frac{a}{L} - 1$	0-b

$$\Delta_{11} = \sum \int M_1 \cdot m_1 \frac{d\alpha}{EI}$$

$$= \int_0^a \left(\frac{Wb}{L} \cdot \alpha \right) \left(1 - \frac{\alpha}{L} \right) \frac{d\alpha}{EI} + \int_0^b \frac{W\alpha}{L} \cdot \alpha \left(\frac{\alpha}{L} \right) \frac{d\alpha}{EI}$$

$$= \frac{W a^3 b}{6L^3 EI} + \frac{W a b^3}{3L^3 EI}$$

$$\Delta_{21} = \sum \int M_1 \cdot m_2 \frac{d\alpha}{EI}$$

$$= -\frac{W a^3 b}{3L^3 EI} - \frac{W a b^3}{6L^3 EI}$$

$$\delta_{11} = \sum \int m_1^2 \frac{d\alpha}{EI} = \int_0^L \left(1 - \frac{\alpha}{L} \right)^2 \frac{d\alpha}{EI}$$

(NO. force in b/w A & C such that the portion to be broken into two parts)
for m_1 system.

$$= \frac{L}{3EI}$$

$$\delta_{22} = \sum \int m_2^2 \frac{d\alpha}{EI} = \int_0^L \left(\frac{\alpha}{L} \right)^2 \frac{d\alpha}{EI}$$

$$= \frac{L}{3EI}$$

$$\delta_{12} = \delta_{21} = \int_L m_1 m_2 \frac{d\alpha}{EI}$$

$$= \int_0^L \left(1 - \frac{\alpha}{L} \right) \left(-\frac{\alpha}{L} \right) \frac{d\alpha}{EI}$$

$$= \frac{-L}{6EI}$$

$$\delta_{11} x_1 + \delta_{12} x_2 + \Delta_{11} = 0$$

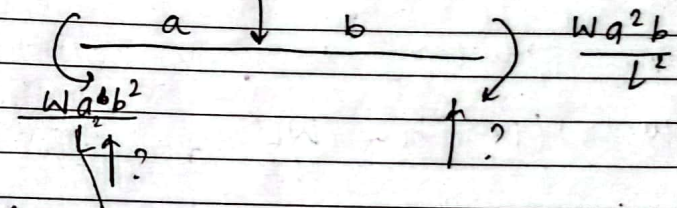
$$\delta_{21} x_1 + \delta_{22} x_2 + \Delta_{21} = 0$$

$$x_1 = -\frac{W a b^2}{L^2}$$

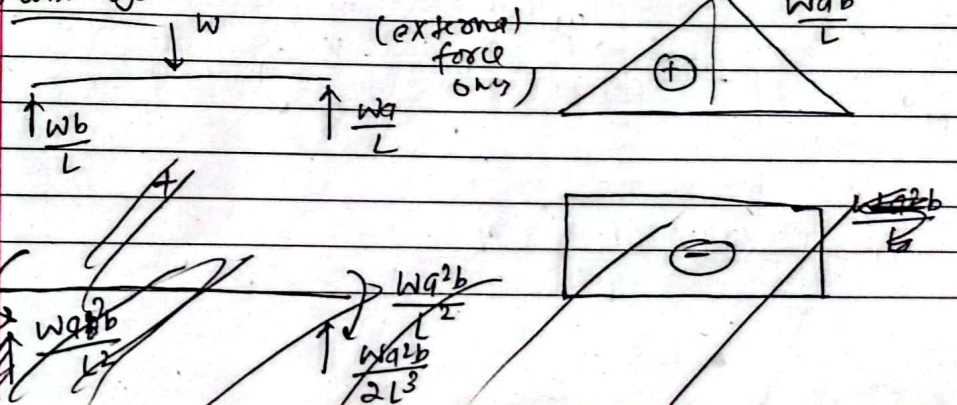
$$x_2 = +\frac{W a^2 b}{L^2}$$

Remember

Fixed end moment (FEM)

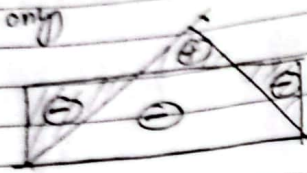


For whole system



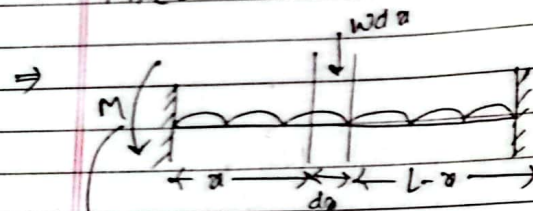
$\frac{wab^2}{L}$
 $\frac{wa^3}{L}$

 Redundant force only

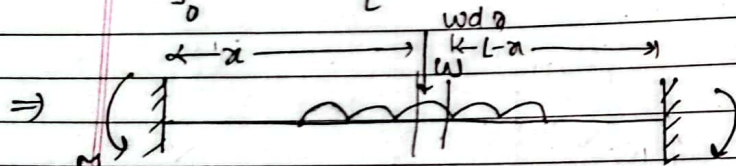


superimposed

* Fixed end Moment (FEM)

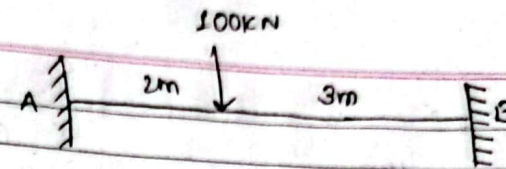


$$\int_0^L \frac{(w \cdot da)(L-a)^2 a}{L^2} = \frac{WL^2}{12}$$

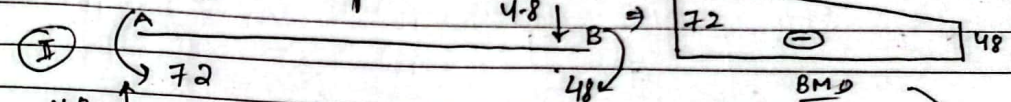
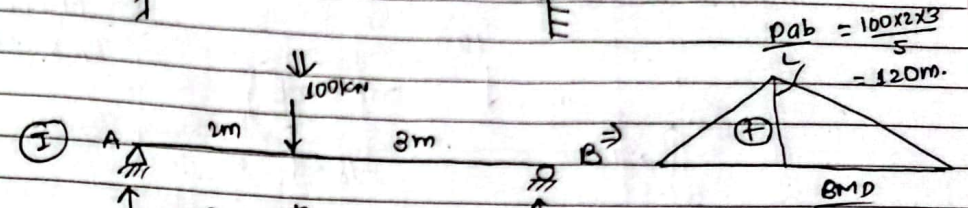


$$M = \int_0^L \frac{(w da)(L-a)^2 a}{L^2}$$

Q

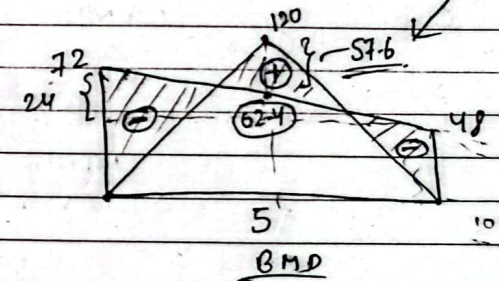


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$$M_A = - \frac{Pab^2}{L^2} = - \frac{100 \times 2 \times 3^2}{5^2} = -72 \text{ kNm}$$

$$M_B = + \frac{Pa^2b}{L^2} = + \frac{100 \times 2^2 \times 3}{5^2} = +48 \text{ kNm}$$



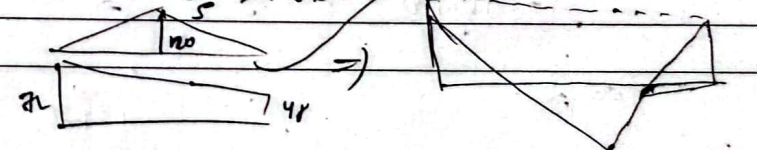
→ unbalanced end moment

$$= 72 - 48$$

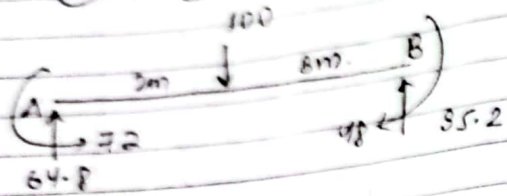
$$= 24 \text{ ACW}$$

So, \$24\text{ clw}\$ moment to be provided at ~~end~~
 by putting forces at
 end to get \$24\text{ clw}\$
 moment.

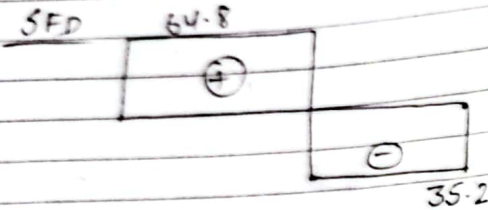
$$\text{each side } \frac{24}{2} = 12 \text{ kN}$$



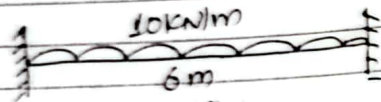
Thus, (I) + (II)



Check: $\sum M_B = 0$
 $R_A \times 5 - 72 - 100 \times 5 = -48$
 $R_A = 64.8$

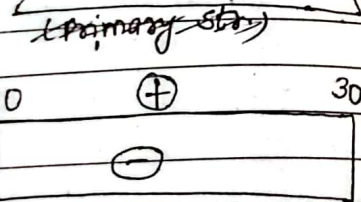
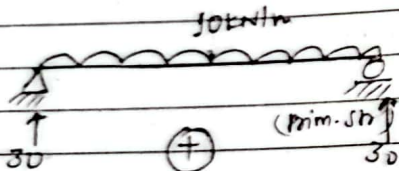


Q.



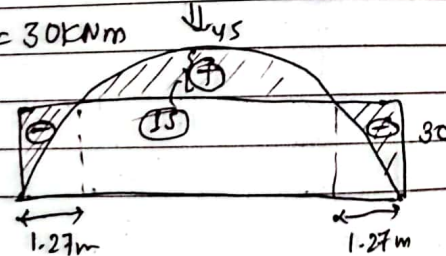
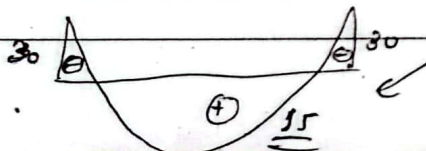
$\frac{WL^2}{8}$

$45 \text{ kNm} - 300 - 50^2$

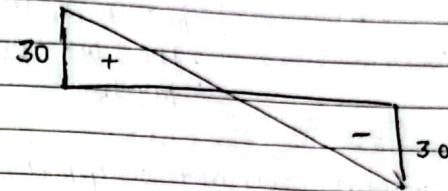
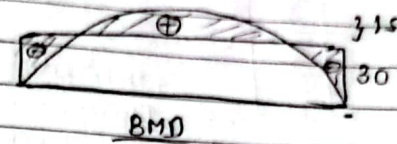
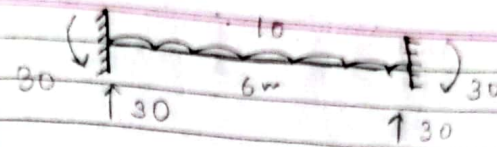


$\frac{WL^2}{12} = \frac{10 \times 6^2}{12}$
 $= 30 \text{ kNm}$

$\frac{WL^2}{12} = 30 \text{ kNm}$



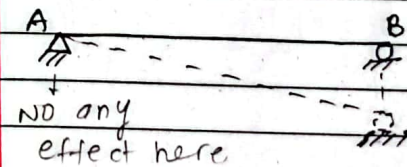
Overall $\frac{1}{8}$



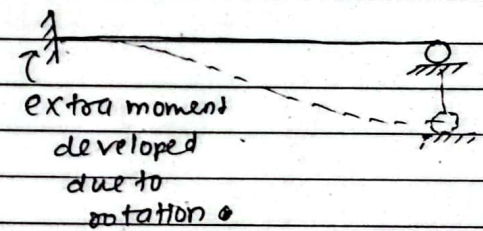
Effect of support settlement and rotation:-

Determine str. & settlement or rotation of
 जहाँ extra load/moment आयेगा ??

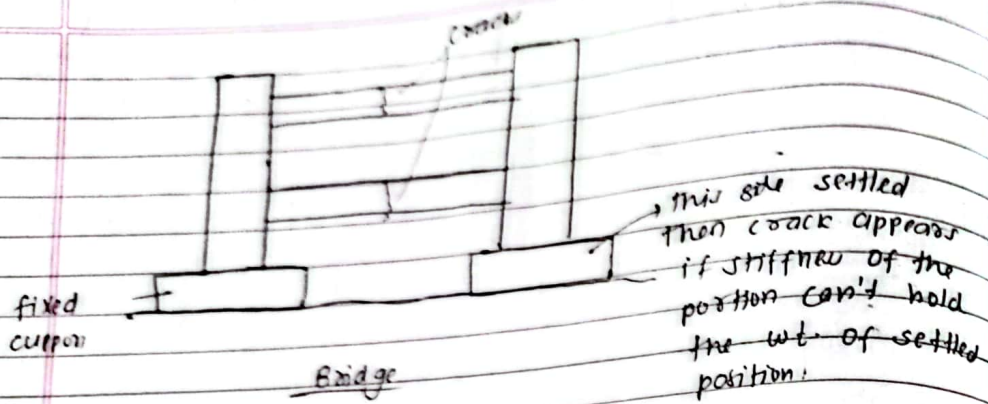
But, for indeterminate str. & settlement or
 rotation of जहाँ extra load/moment आयेगा



Determinate

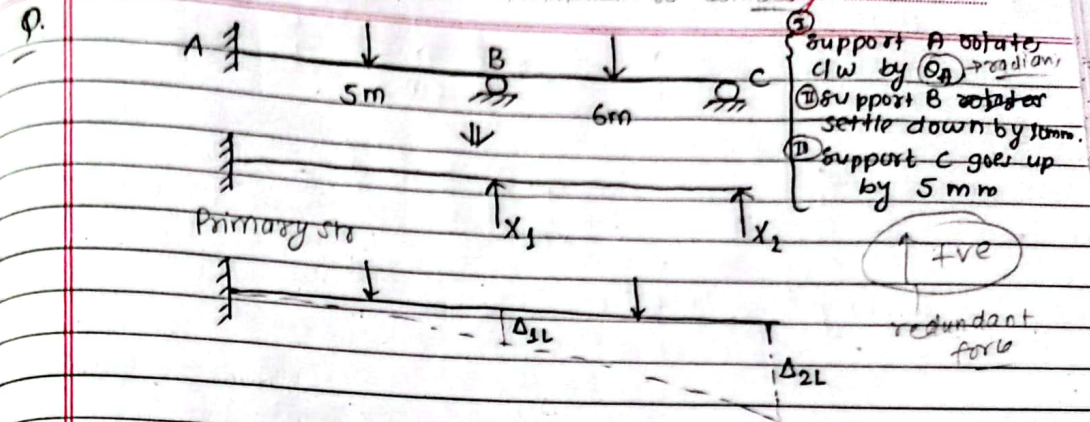


Indeterminate

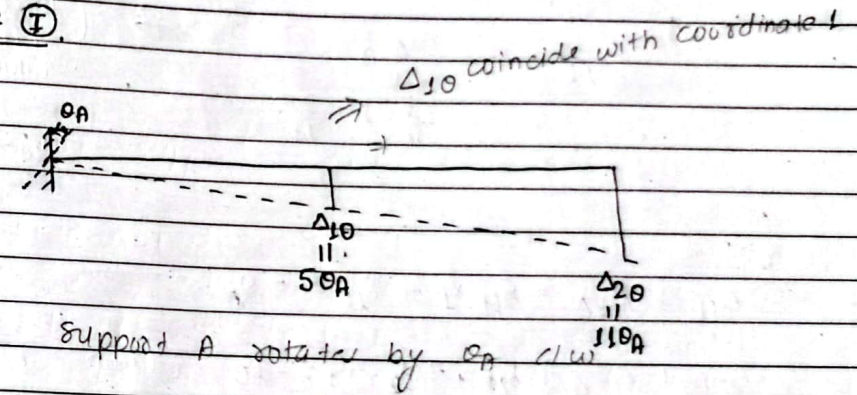


→ unequal settlement (if beam cannot support the load due to unequal settlement then crack formation occurs).

if redundant motion can't settlement and Δ_{10} coincide
Here, x_1 & x_2 at position of settlement & coincide



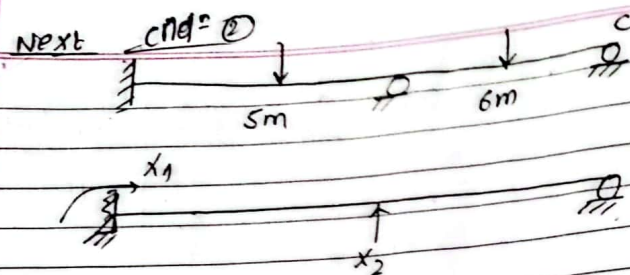
cond = (1)



Support rot = θ_A consider Δ_{10} coincide with coordinate 1
If rot or settlement coincide or no effect on other eq.
only affect net disp. on that coordinate

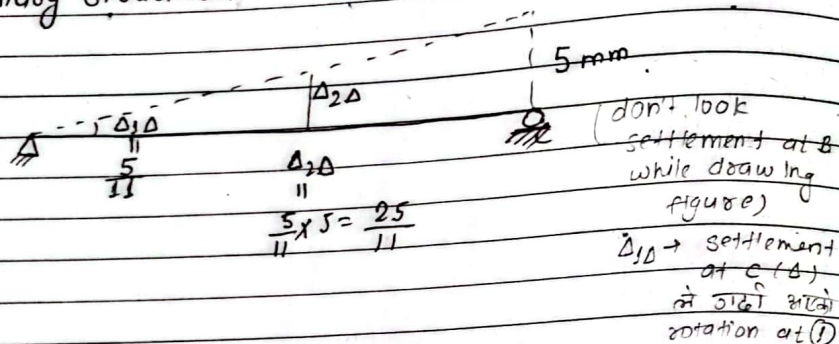
$$\text{Now, } \Delta_{1L} + \Delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = \Delta_1 = -10 \quad \uparrow \text{ve}$$

$$\Delta_{2L} + \Delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = \Delta_2 = 5$$



$\theta_A \rightarrow$ coincide with ①
 $\Delta_B \rightarrow$ coincide with ②
 $\Delta_C \rightarrow$ do not coincide.

Primary structure:

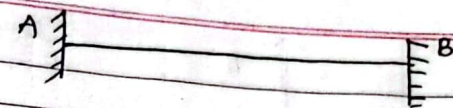


Eqn:-

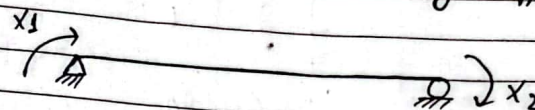
$$\Delta_{1L} + \Delta_{1D} + \delta_{11} X_1 + \delta_{12} X_2 = \theta_A$$

$$\Delta_{2L} + \Delta_{2D} + \delta_{21} X_1 + \delta_{22} X_2 = -10 \quad (\text{down settlement -ve})$$

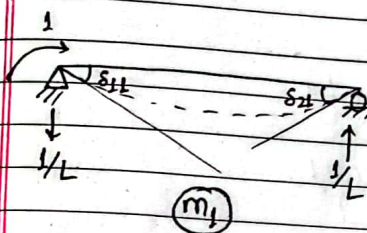
(9)



Support A rotates by θ_A .



For $X_1 = 1 \text{ kNm}$



$$\delta_{11} = \frac{L}{3EI}$$

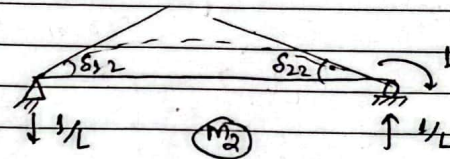
$$\delta_{21} = \frac{L}{6EI}$$

$$\delta_{22} = \frac{L}{3EI}$$

$$\delta_{11} X_1 + \delta_{12} X_2 = \theta_A$$

$$\delta_{21} X_1 + \delta_{22} X_2 = 0$$

For $X_2 = 1$

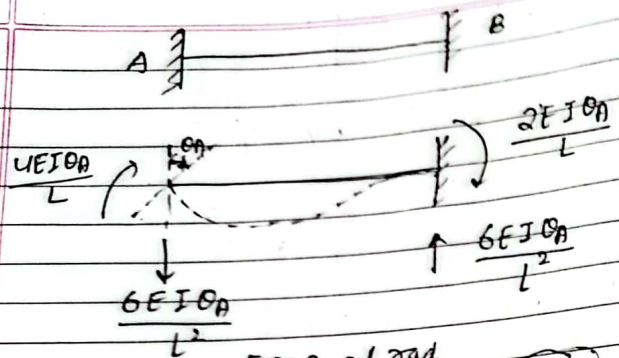


$$\frac{L}{3EI} X_1 - \frac{L}{6EI} X_2 = \theta_A \quad \text{--- (I)}$$

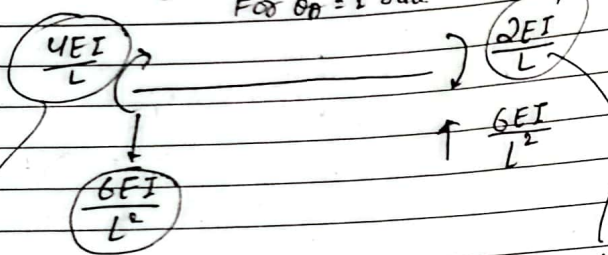
$$-\frac{L}{6EI} X_1 + \frac{L}{3EI} X_2 = 0 \quad \text{--- (II)}$$

$$X_1 = \frac{4EI\theta_A}{L}, \quad X_2 = \frac{2EI\theta_A}{L}$$

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For $\theta_A = 1 \text{ rad}$



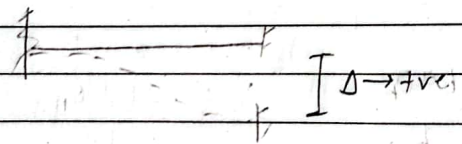
A मा आये rotation को भाई pt. B मा rotation आये.
 → rotation आये support में moment
 → वही support rotate आये से चलाता वह moment आये, चलाते 1/2 moment आये support में आये.

$\left(\frac{1}{2}\right)$ is called carry-over factor.

Sign convention

Clw +ve

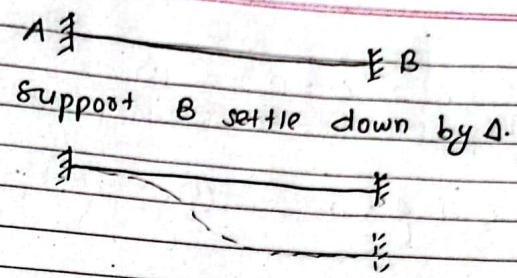
Settlement: left support up & right support down +ve



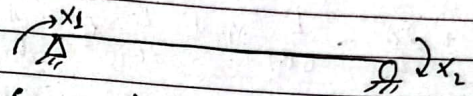
Sign convention

Settlement: left support up & right support down $\Rightarrow$ (+ve)
 Rotation: Clw rotation = (+ve)

(9)



Prim. str.



Flexibility coeff

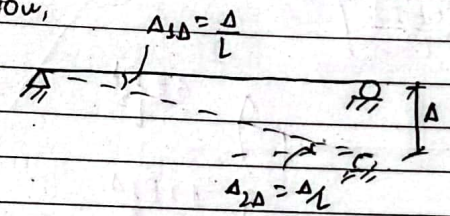
$$\delta_{11} = \frac{L}{3EI}$$

$$\delta_{12} = \delta_{21} = -\frac{L}{6EI}$$

$$\delta_{22} = \frac{L}{3EI}$$

$\Delta_{1\Delta} \rightarrow$ pt. 1 मा आये displacement due to settlement Δ

Now,



eq^s are

$$\delta_{11}x_1 + \delta_{12}x_2 + \Delta_{1\Delta} = 0$$

$$\delta_{11}x_1 + \delta_{12}x_2 + \frac{\Delta}{L} = 0 \quad \text{--- (1)}$$

$$\delta_{21}x_1 + \delta_{22}x_2 + \Delta_{2\Delta} = 0$$

$$\text{On } \delta_{21}x_1 + \delta_{22}x_2 + \frac{\Delta}{L} = 0 \quad \text{--- (2)}$$

$\Delta_{1\Delta} = \Delta$ from area under x_2 is moment & Δ is settlement so no coincide.
 If x_2 vertical reduction force आये $\Delta_{2\Delta} = \Delta$

From ①

$$\frac{L}{3EI} \cdot X_1 - \frac{L}{6EI} X_1 + \frac{\Delta}{L} = 0 \quad \text{--- (3)}$$

From ②

$$-\frac{L}{6EI} X_1 + \frac{L}{3EI} X_2 + \frac{\Delta}{L} = 0 \quad \text{--- (4)}$$

eqⁿ ① $\times 2$ + eqⁿ ②

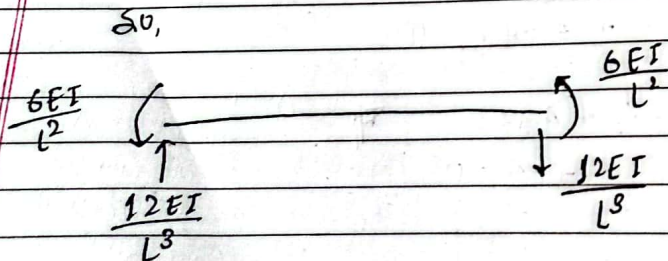
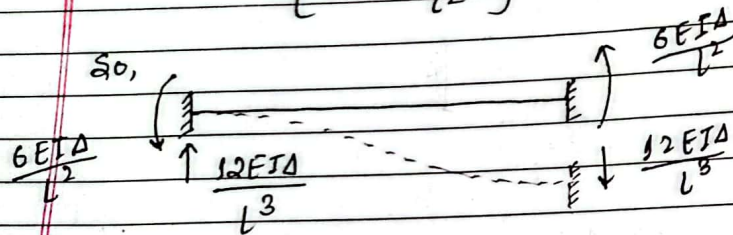
$$\frac{2L}{3EI} X_1 - \frac{L}{3EI} X_1 + \frac{2\Delta}{L} - \frac{L}{6EI} X_1 + \frac{L}{3EI} X_2 + \frac{\Delta}{L} = 0$$

$$\therefore \left[X_1 = -\frac{6EI\Delta}{L^2} \right]$$

& From ③

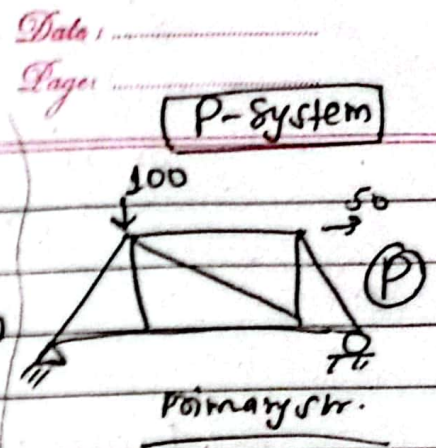
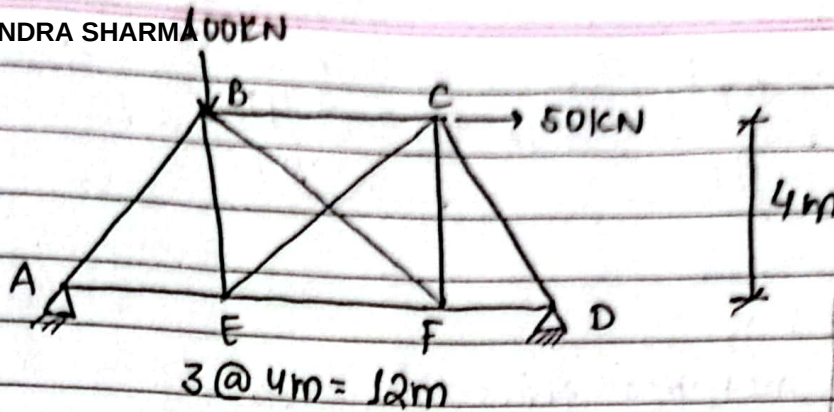
$$\frac{L}{3EI} \left(-\frac{6EI\Delta}{L^2} \right) + \frac{\Delta}{L} = \frac{L}{6EI} X_2$$

$$\therefore \left[X_2 = -\frac{6EI\Delta}{L^2} \right]$$



A TRUSS

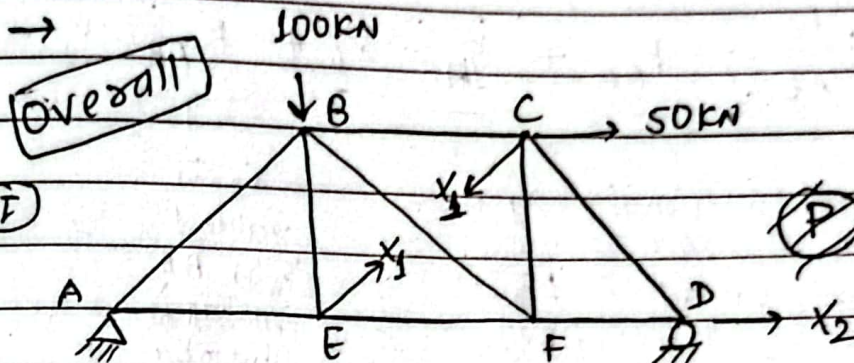
SURENDRA SHARMA



$DSI = 1 + 1 = 2$
(compatibility should be solved for 2 unknowns)

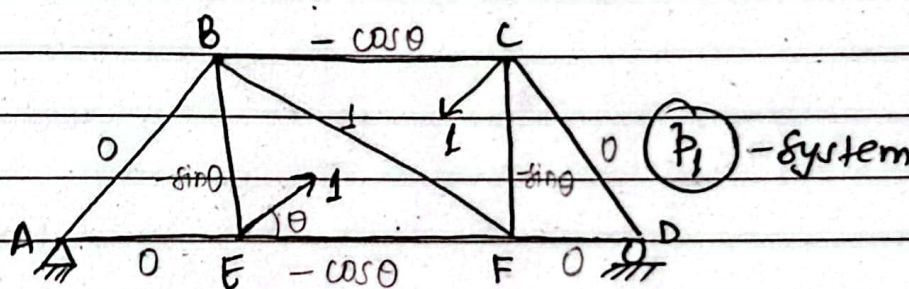
(I) → Primary str. remove extra member force & extra rxn.

& assign unknown or coordinate



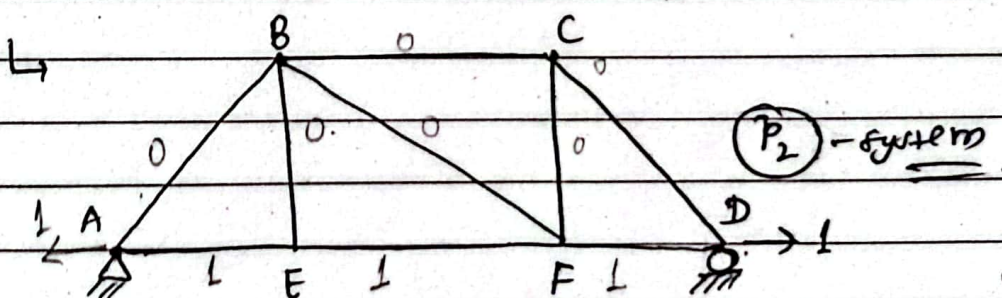
4. Primary structure with co-ordinate rxn & member force (x_1 & x_2)

• Structure with unit redundant force:-



(II) Unit force diagram (for unit value of redundant forces)

(III) Find member forces in each case P , P_1 & P_2 .



(IV) Find
 $\Delta_{1L} = \frac{\sum P P_1 L}{AE}$
 $\Delta_{2L} = \frac{\sum P P_2 L}{AE}$

(VI) Find other member forces

(V) Compatibility eqn. & solve redundant force

Compatibility eqns.

$$\delta_{11} X_1 + \delta_{12} X_2 + \Delta_{1L} = 0$$

$$\delta_{21} X_1 + \delta_{22} X_2 + \Delta_{2L} = 0$$

Solve to find
Redundant
forces

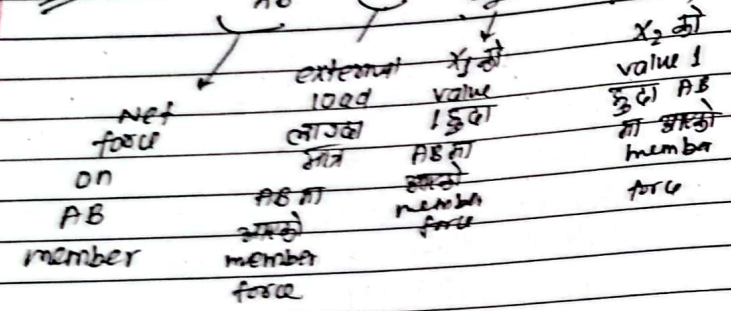
where,

$$\delta_{11} = \frac{\sum P_1^2 L}{AE}, \quad \delta_{12} = \delta_{21} = \frac{\sum P_1 P_2 L}{AE}, \quad \delta_{22} = \frac{\sum P_2^2 L}{AE}$$

→ Find X_1 & X_2

→ Finding member forces
Technique

PSP. → $P'_{AB} = P_{AB} + \frac{P_{AB}}{AE} X_1 + \frac{P_{AB}}{AE} X_2$



Effect of temperature variation & misfit:-
If temperature variation and misfit are in some members, are given, it produces extra forces in all members.

If temperature variation and misfit coincide with co-ordinates, then these values are written on the right side of compatibility equation as net displacement at that coordinate.

If temperature variation and misfit do not coincide with co-ordinate, then displacement at co-ordinates are written on left side of compatibility equation.

eg:- $\delta_{11} X_1 + \delta_{12} X_2 + \Delta_{1L} = \underbrace{(L \alpha t)}_{\text{say } +20} + \underbrace{10}_{\text{say 10mm too long}}$

While writing on right side

coincide case
EC member in tension

Temp ↑ & too long ⇒ (-ve) → coincide are तेड़ी member
Temp ↑ & too short ⇒ (+ve) →

too long → member लम्बित पायेगा so लो
corr. value ले compress कर पार्गे
so (-ve)

too short → member खुसिया पार्गे so लो
corr. value ले member elongate
कर पार्गे so (+ve)

coincide are तेड़ी member का मत्र effect हुआ, coincide मत्र खसका हुआ

CLT
1st

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Not coincide case

→ If temperature variation and misfit is given in member EF

Left side of member EF

$$\left. \begin{aligned} \Delta_{1f} &= \delta_{1f} + \epsilon_f \\ \Delta_{2f} &= \delta_{2f} + \epsilon_f \end{aligned} \right\} \text{Here } \epsilon_f = \text{temp. var. of steel in both } 1 \text{ \& } 2 \text{ coordinate}$$

$$\Delta_{1f} = \underbrace{\delta_{1f}}_{\text{misfit}} + \epsilon_f$$

$$\Delta_{2f} = \delta_{2f} + \epsilon_f$$

Thus eqⁿ formed,

$$\delta_{1f} x_1 + \delta_{2f} x_2 + \underbrace{\Delta_{1f} + \Delta_{2f}}_{\substack{\text{coincide value} \\ \text{value on left side}}} = \Delta_1$$

Here,

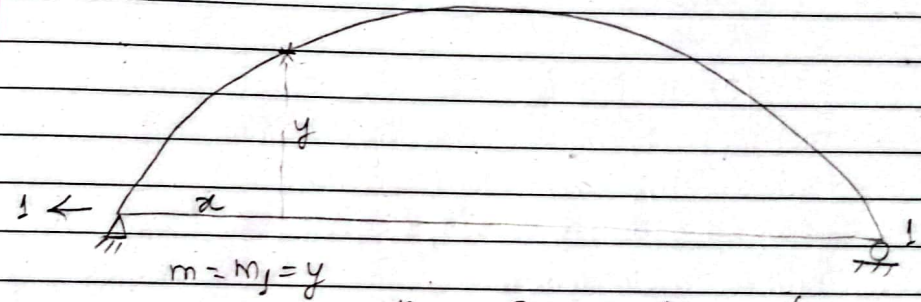
too long +
 too short -
 temp inc. +
 temp dec. -

* Two-Hinged Arch

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[DST=1]

Primary structure

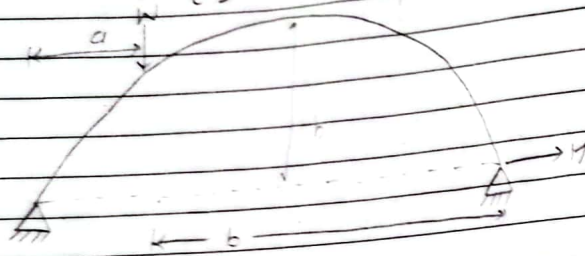


$$\Delta_{1L} = \int \frac{M_L \cdot m_1 ds}{EI} = \int \frac{M_{beam} \cdot m_1 ds}{EI}$$

$$\delta_{11} = \int \frac{m_1^2 ds}{EI}$$

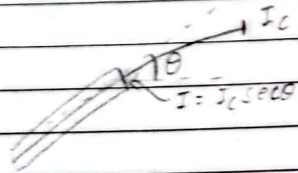
$$\delta_{11} x_1 + \Delta_{1L} = 0$$

$$\Delta_{II} = \sum \frac{M_b m ds}{EI} = \sum \frac{M_b y ds}{EI}$$



$$H = - \frac{\int \frac{M_b y ds}{EI}}{\int \frac{y^2 ds}{EI}}$$

m-y



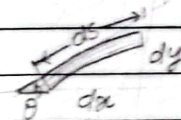
$$[I = I_0 \sec \theta]$$

Moment of inertia at any section

θ = slope at that section

I_0 → ground or fixed I

consider small length.



$$\frac{ds}{dx} = \sec \theta$$

$$[ds = dx \sec \theta]$$

$\int$ of that section

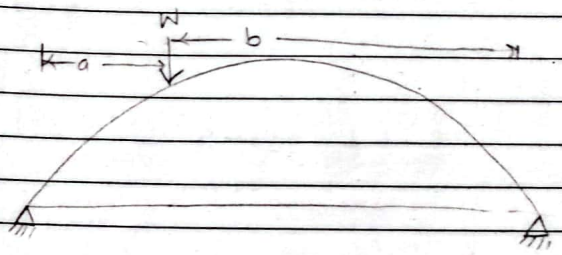
$$H = - \frac{\int \frac{M_b y ds}{EI}}{\int \frac{y^2 ds}{EI}}$$

$$= - \frac{\int \frac{M_b y dx \sec \theta}{EI \sec \theta}}{\int \frac{y^2 dx \sec \theta}{EI \sec \theta}}$$

Thus,

$$[H = - \frac{\int M_b y dx}{\int y^2 dx}]$$

Lab Formula



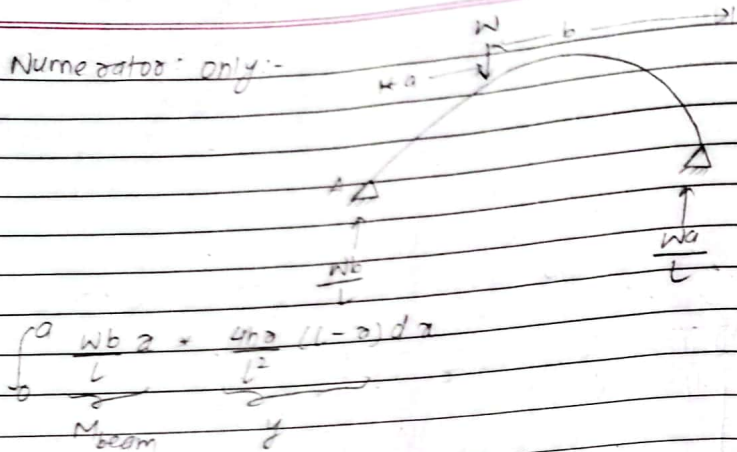
Parabolic arch

$$y = \frac{4h\alpha}{L^2} (L-\alpha)$$

$$\int y^2 d\alpha = \int \left[\frac{4h\alpha}{L^2} (L-\alpha) \right]^2 d\alpha$$

$$\text{Denominator} = \left[\int y^2 d\alpha = \frac{8}{15} h^2 L \right] \neq$$

Number of atoms: only:-



$$= \frac{Wh a (1-a) (L^2 + aL - a^2)}{2L^2}$$

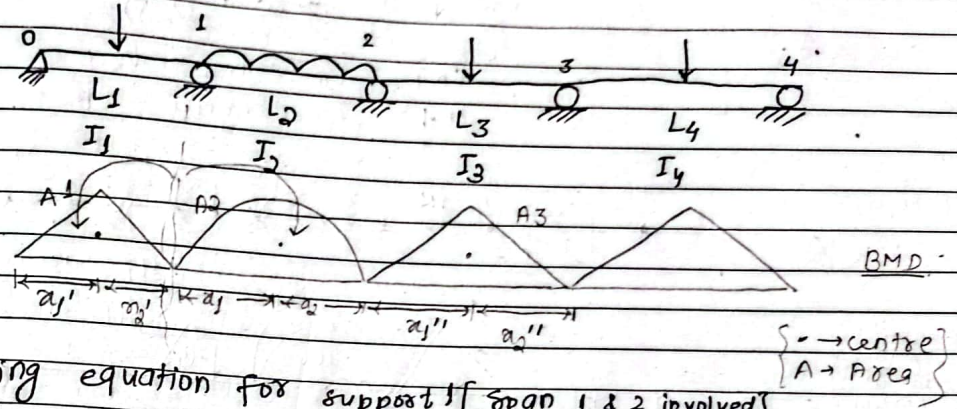
Thus,

$$H = \frac{5W_0 a (1-a)(L^2 + aL - a^2)}{8h\beta^3}$$

(L2) Put $a = nL$ } $n \rightarrow$ factor ($n \leq L$)

$$-5Wn \left\{ H = \frac{5WnL}{8h} [1 - 2n^2 + n^3] \right\}$$

Three Moment Equation



Writing equation for support 1 [Span 1 & 2 involved]

$$\left\{ M_0 \cdot \frac{L_1}{I_1} + 2 M_1 \left[\frac{L_1}{I_1} + \frac{L_2}{I_2} \right] + M_2 \cdot \frac{L_2}{I_2} = \right.$$

$$-6 \left[\frac{A_1 \cdot A_2 \cdot \alpha_1}{L_2 I_2} + \frac{A_3 \cdot \alpha_1'}{L_3 I_3} \right]$$

(x_1', x_2) Opposite of centre

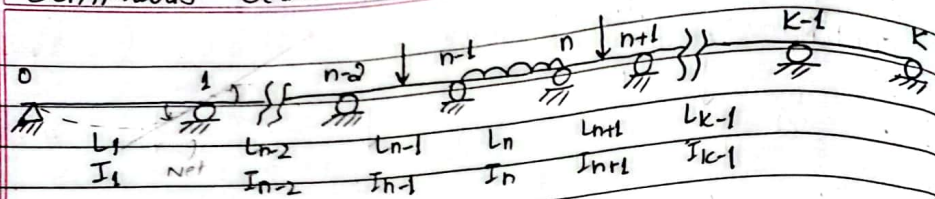
while waiting equation for support (2) {span 2 & 3 involved}

$$M_1 \cdot \frac{L_2}{I_2} + 2M_2 \left[\frac{L_2}{I_2} + \frac{L_3}{I_3} \right] + M_3 \cdot \frac{L_3}{I_3} = -6 \left[\frac{A_2 \cdot x_1}{L_2 I_2} + \frac{A_3 \cdot x_2''}{L_3 I_3} \right]$$

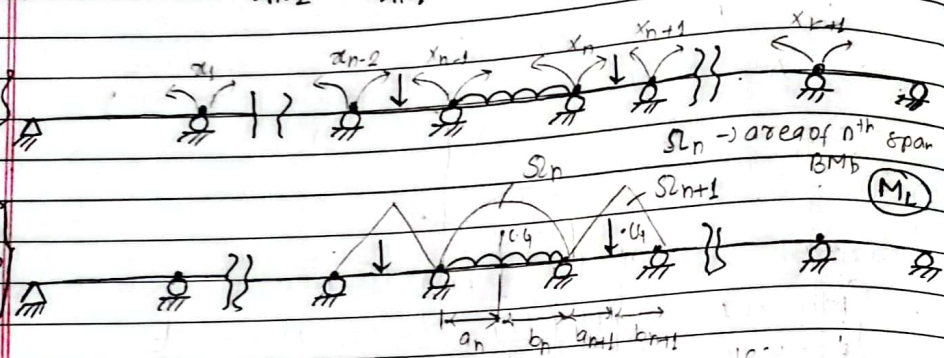
Continuous beam

(primary)

x_2 moment released (due to internal hinges)

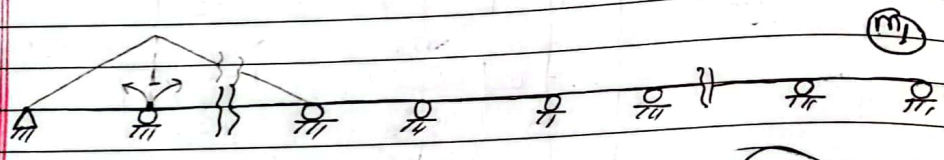


Primary str.

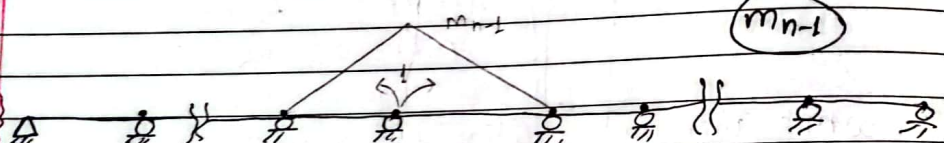


Simply supported beam w/o redundant force with external force lib BMD

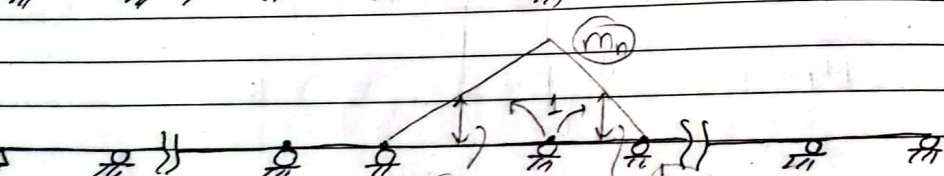
For unit moment at 1



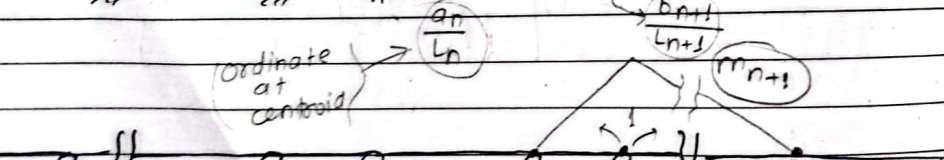
For unit moment at m_{n-1}



For unit moment at m_n



For unit moment at m_{n+1}



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Compatibility equation of support 1

$$\delta_{11} X_1 + \delta_{12} X_2 + \dots + \delta_{1(n-1)} X_{n-1} + \delta_{1n} X_n + \delta_{1(n+1)} X_{n+1} + \dots + \delta_{1,k-1} X_{k-1} + \Delta_{1L} = 0$$

Net rotation at support 1 is zero

Similarly
Compatibility equation of support n:-

$$\delta_{n1} X_1 + \delta_{n2} X_2 + \dots + \delta_{n,n-1} X_{n-1} + \delta_{nn} X_n + \delta_{n,n+1} X_{n+1} + \dots + \delta_{n,k-1} X_{k-1} + \Delta_{nL} = 0 \quad \text{--- (1)}$$

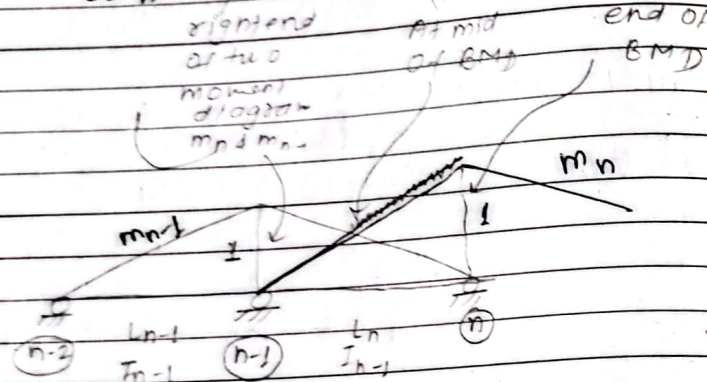
$$\delta_{n1} = \frac{\int m_n m_1 da}{EI} \quad \text{or use Graphical method.}$$

$$\int_{AB} \frac{M_L m_1}{EI} da \quad \text{portion taken} \rightarrow \text{value not 0}$$

$\delta_{n1} \rightarrow n \neq 1$ portion

In this equation all other coefficients except $\delta_{n,n-1}$; δ_{nn} ; $\delta_{n,n+1}$ & Δ_n all other coefficients are zero
(Find these and substitute to get value)
(Use SFD or MFD or both)

$$\delta_{n,n-1} = \frac{L_n}{6EI_n} \left[1 \times 0 + 4 \cdot \frac{1}{2} \cdot \frac{1}{2} + 0 \cdot 1 \right] = \frac{L_n}{6EI_n}$$



→ n & $n-1$ are common BMD portion for both

→ common part for all BMD components zero SFD

$$\delta_{nn} = \frac{L_n}{6EI_n} \left[0 + 4 \cdot \frac{1}{2} \times \frac{1}{2} + 1 \times 1 \right] + \frac{L_{n+1}}{6EI_{n+1}} \left[1 \times 1 + 4 \cdot \frac{1}{2} \times \frac{1}{2} + 0 \times 1 \right]$$

$$= \frac{L_n}{3EI_n} + \frac{L_{n+1}}{3EI_{n+1}}$$

$$\delta_{n,n+1} = \frac{L_{n+1}}{6EI_{n+1}} \left[1 \times 0 + 4 \cdot \frac{1}{2} \times \frac{1}{2} + 0 \times 1 \right] = \frac{L_{n+1}}{6EI_{n+1}}$$

Now,

$$\frac{\int M \cdot m \, dx}{EI}$$

Area $\times$ ordinate

$$\Delta_n = \frac{S_n \cdot \frac{a_n}{L_n}}{EI_n} + \frac{S_{n+1} \cdot \frac{b_{n+1}}{L_{n+1}}}{EI_{n+1}}$$

Area of $\frac{M}{EI}$ diagram for unit moment at n

ordinate at centroid

Thus, From eqⁿ (1)

$$\frac{L_n}{6EI_n} \times X_{n-1} + \left(\frac{L_n}{3EI_n} + \frac{L_{n+1}}{3EI_{n+1}} \right) X_n + \frac{L_{n+1}}{6EI_{n+1}} \times X_{n+1} + \frac{S_n \cdot \frac{a_n}{L_n}}{EI_n} + \frac{S_{n+1} \cdot \frac{b_{n+1}}{L_{n+1}}}{EI_{n+1}} = 0$$

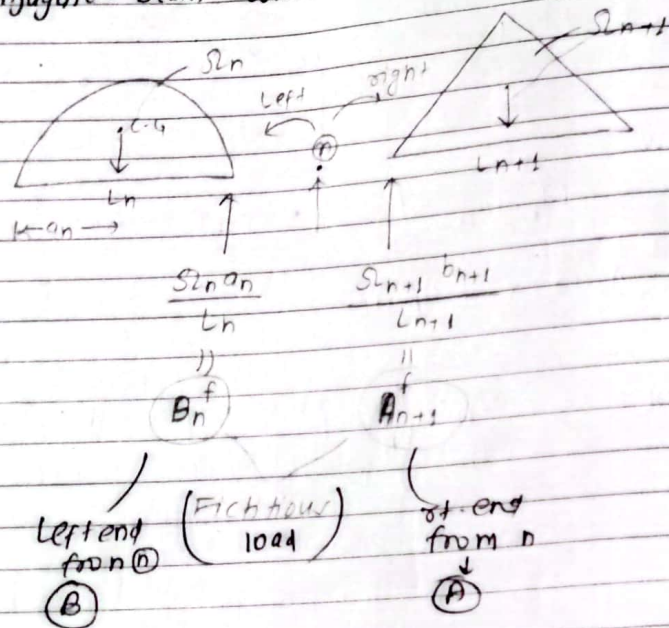
$$\text{or, } \frac{L_n}{I_n} X_{n-1} + 2 \left[\frac{L_n}{I_n} + \frac{L_{n+1}}{I_{n+1}} \right] X_n + \frac{L_{n+1}}{I_{n+1}} X_{n+1} = -6 \left[\frac{S_n \cdot \frac{a_n}{L_n}}{I_n} + \frac{S_{n+1} \cdot \frac{b_{n+1}}{L_{n+1}}}{I_{n+1}} \right]$$

offering support at node n

→ This is three moment equation.

(X)

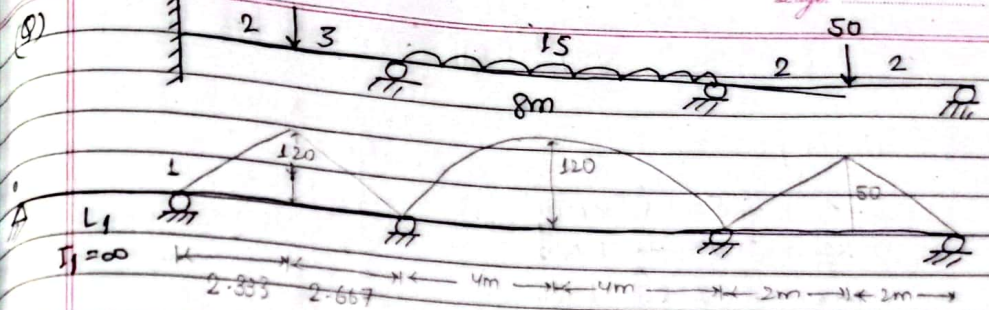
Conjugate beam with BMD as load



Thus,

$$\frac{L_n}{I_n} X_{n-1} + 2 \left[\frac{L_n}{I_n} \cdot \frac{S_n}{L_n} + \frac{L_{n+1}}{I_{n+1}} \right] X_n + \frac{L_{n+1}}{I_{n+1}} \cdot \frac{S_{n+1}}{L_{n+1}} = -6 \left[\frac{B_n^f}{I_n} + \frac{A_{n+1}^f}{I_{n+1}} \right]$$

This is other form of three moment equation.



Position of C.G. of 1st = $\frac{1}{2} \times 2 \times 120 \times 2 \times \frac{2}{3} + \frac{1}{2} \times 3 \times 120 \times \left(2 + \frac{2}{3}\right) + \frac{1}{2} \times 5 \times 120$

$$= 2.333m$$

Support 1

$$M_0 \cdot \frac{L_1}{\infty} + 2 \left[\frac{L_1}{\infty} + \frac{5}{I} \right] M_1 + \frac{5}{I} M_2 = -6 \left[0 + \frac{300 \times 2.333}{5I} \right]$$

$$\text{or, } 10M_1 + 5M_2 = -960.12 \quad \text{--- (1)}$$

Support 2

$$M_1 \cdot \frac{5}{I} + 2 \left[\frac{5}{I} + \frac{8}{I} \right] M_2 + \frac{8}{I} M_3 = -6 \left[\frac{300 \times 2.333}{5I} + \frac{640 \times 4}{8I} \right]$$

$$\text{or, } 5M_1 + 26M_2 + 8M_3 = -2759.88 \quad \text{--- (2)}$$

Support 3

④ → No moment at support ④

$$M_2 \cdot \frac{8}{I} + 2 \cdot M_3 \left[\frac{8}{I} + \frac{4}{I} \right] + \frac{4}{I} M_4 = -6 \left[\frac{100 \times 4 \times 4}{8I} + \frac{100 \times 2}{4I} \right]$$

$$\text{or, } 8M_2 + 24M_3 + 0 = -2220 \quad \text{--- (3)}$$

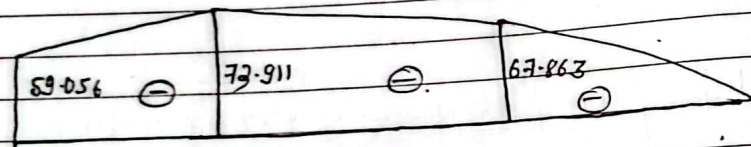
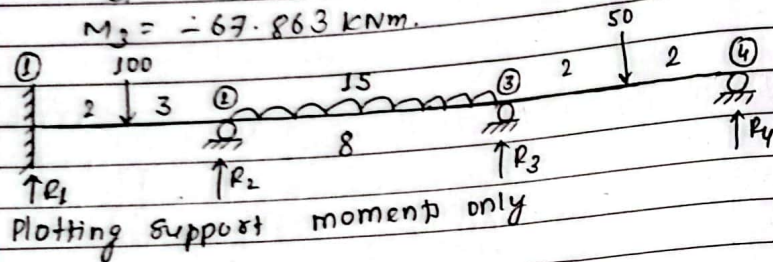
But if overhanging at end.
 $(M_y = -100)$

Solving,

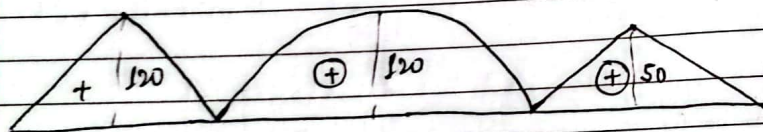
$$M_1 = -59.056 \text{ kNm}$$

$$M_2 = -73.911 \text{ kNm}$$

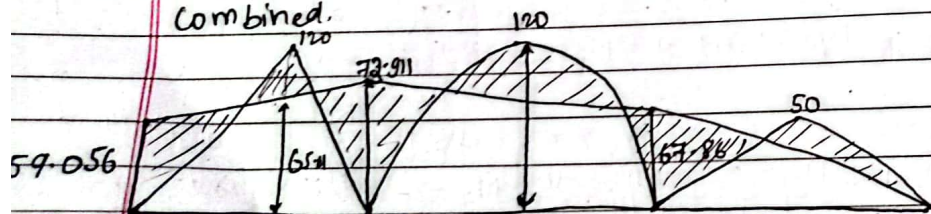
$$M_3 = -67.863 \text{ kNm}$$



Due to external reaction



Combined.



Now finding support reactions:-

$$M_2 = -73.911$$

$$\text{or, } R_1 \times 5 - 59.056 - 100 \times 3 = -73.911$$

$$\Rightarrow [R_1 = 57.03 \text{ kN}]$$

$$M_3 = -67.86$$

$$\text{or, } 57.03 \times 12 - 100 \times 12 - 59.056 + R_2 \times 8 - \frac{15 \times 8 \times 8}{2} = -67.86$$

$$\Rightarrow [R_2 = 103.72 \text{ kN}]$$

$$M_3 = -67.86$$

$$\text{or, } R_4 \times 4 - 50 \times 2 = -67.86$$

$$\Rightarrow [R_4 = 8.035 \text{ kN}]$$

$$M_2 = -73.911$$

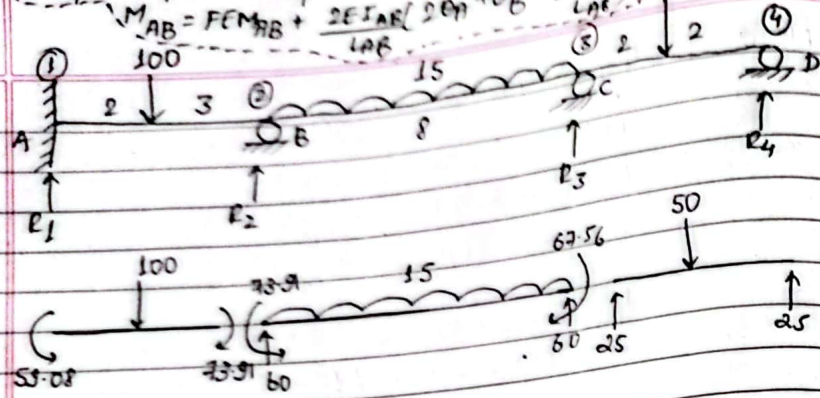
$$\text{or, } 8.035 \times 12 + R_3 \times 8 - 50 \times 10 - 15 \times 8 \times 4 = -73.911$$

$$\Rightarrow [R_3 = 101.21 \text{ kN}]$$

$$\text{check, } R_1 + R_2 + R_3 + R_4 = 100 + 15 \times 8 + 50$$

Q.

Slope deflection equation:



→ Here,

$$FEM_{AB} = -72 \text{ kNm}$$

$$FEM_{BA} = 48 \text{ kNm}$$

$$FEM_{BC} = \frac{15 \times 8^2}{12} = -80 \text{ kNm}$$

$$FEM_{CB} = 80 \text{ kNm}$$

$$FEM_{CD} = -25 \text{ kNm}$$

$$FEM_{DC} = 25 \text{ kNm}$$

Now,

$$M_{AB} = FEM_{AB} + \frac{2EI}{L} (2\theta_A + \theta_B - \frac{3\Delta_{AB}}{L})$$

$$= -72 + \frac{2EI}{5} (2 \times 0 + \theta_B)$$

$$M_{AB} = -72 + 0.4\theta_B EI \quad \text{--- (1)}$$

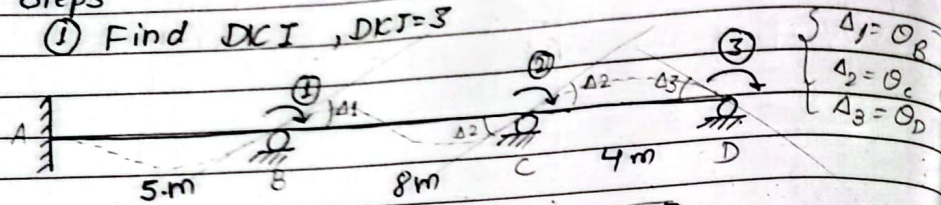
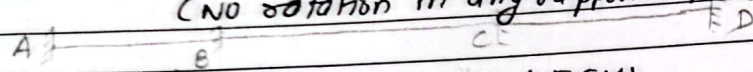
$$M_{BA} = 48 + \frac{2EI}{5} (2 \times \theta_B + 0 - 0) = 48 + 1.6\theta_B EI$$

$$M_{BC} = -80 + \frac{2EI}{8} (2\theta_B + \theta_C - 0) = -80 + 0.5\theta_B EI + 0.25\theta_C EI$$

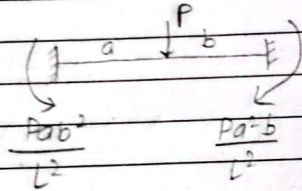
$$M_{CB} = 80 + \frac{2EI}{8} (2\theta_C + \theta_B - 0) = 80 + 0.25\theta_B EI + 0.5\theta_C EI$$

Steps

① Find DCF, DCF=3

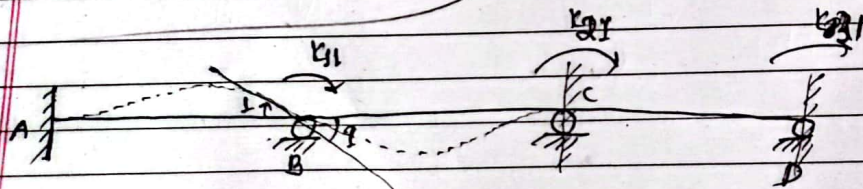
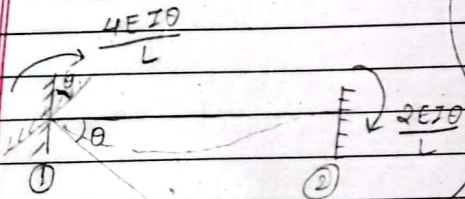
② Draw restrained structure, ~~DCF~~
(No rotation in any support → fixed)

③ Fixed End Moment (FEM)



④ Form stiffness matrix (Force/moment for unit deflection)

0:1 at ①

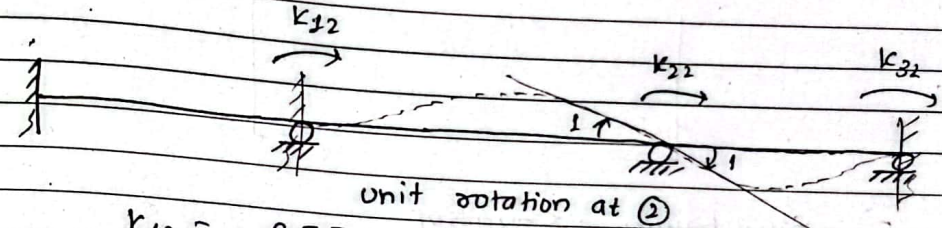


Unit rotation at 1

$$k_{11} = \frac{4EI}{5} + \frac{4EI}{8} \quad [0=1 \text{ at } ①]$$

$$k_{21} = \frac{2EI}{8}$$

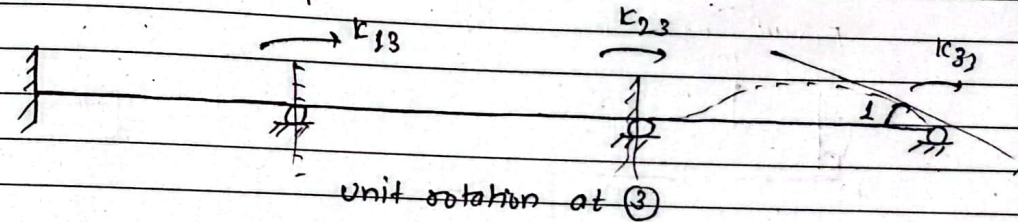
$$k_{31} = 0$$



$$k_{12} = \frac{2EI}{8}$$

$$k_{22} = \frac{4EI}{8} + \frac{4EI}{4}$$

$$k_{32} = \frac{2EI}{4}$$



$$k_{13} = 0$$

$$k_{23} = \frac{2EI}{4}$$

$$k_{33} = \frac{4EI}{4}$$

Thus Stiffness matrix

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

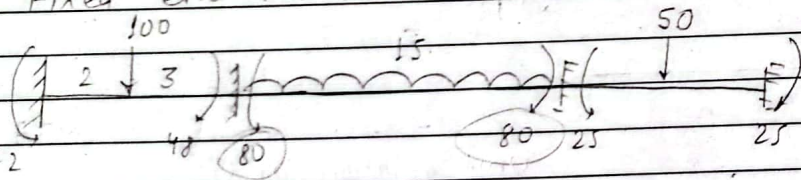
can be formed.

$$\begin{bmatrix} \\ \\ \end{bmatrix}$$

⑤ Equilibrium Equation

$$\begin{cases} k_{11} \Delta_1 + k_{12} \Delta_2 + k_{13} \Delta_3 + P_{1L} = P_1 \\ k_{21} \Delta_1 + k_{22} \Delta_2 + k_{23} \Delta_3 + P_{2L} = P_2 \\ k_{31} \Delta_1 + k_{32} \Delta_2 + k_{33} \Delta_3 + P_{3L} = P_3 \end{cases} \quad (*)$$

Fixed end moments.



$$P_{1L} = 48 - 80 =$$

$$P_{2L} = 80 - 25 =$$

$$P_{3L} = 25$$

+ve → coordinate direction of unit rotation

Here, no moments at any support so, P_1, P_2, P_3 are 0

Solve (*) / equilibrium equations to find

$$\Delta_1 = 0$$

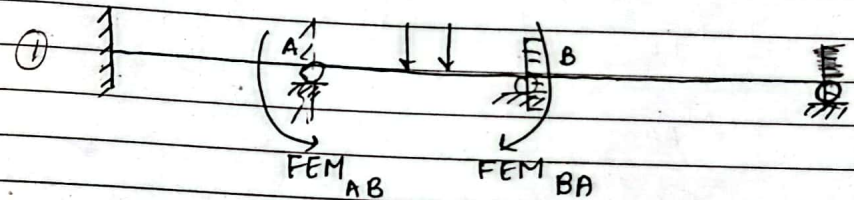
$$\Delta_2 = 0$$

$$\Delta_3 = 0$$

* Use slope deflection equation to find end moments.

* Draw B.M.D.

Slope Deflection Method:



$$\frac{4EI\theta_A}{L}$$

$$\frac{2EI\theta_B}{L}$$

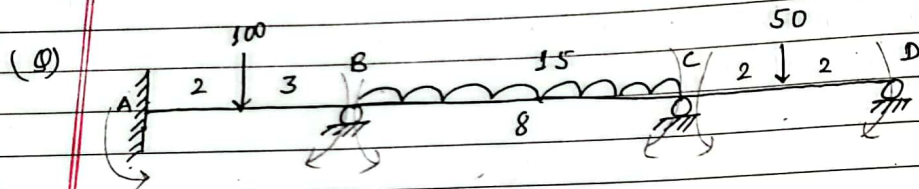
$$\frac{6EI\Delta_{AB}}{L^2}$$

Settlement ले beam को opp dir moment को डालेंगे
 व्यस्तको opp dir moment को डालेंगे

$$M_{AB} = FEM_{AB} + \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta_{AB}}{L^2}$$

$$= FEM_{AB} + \frac{2EI}{L} (2\theta_A + \theta_B - \frac{3\Delta_{AB}}{L})$$

$$M_{BA} = FEM_{BA} + \frac{2EI}{L} (2\theta_B + \theta_A - \frac{3\Delta_{AB}}{L})$$



$$FEM_{AB} = -\frac{100 \times 2^2 \times 3}{5^2} = -72$$

$$FEM_{BA} = \frac{100 \times 2^2 \times 3}{5^2} = 48$$

$$FEM_{BC} = -\frac{15 \times 8^2}{12} = -80$$

$$FEM_{CB} = \frac{15 \times 8^2}{12} = 80$$

$$FEM_{CD} = -\frac{50 \times 2^2 \times 2}{4^2} = -25$$

$$FEM_{DC} = \frac{50 \times 2^2 \times 2}{4^2} = 25$$

Writing slope deflection eq. Method:

$$M_{AB} = FEM_{AB} + \frac{2EI}{L} (2\theta_A + \theta_B - \frac{3\Delta_{AB}}{L})$$

$$\text{or, } M_{AB} = -72 + \frac{2EI}{5} (0 + \theta_B - 0) \quad \text{--- (i)}$$

$$M_{BA} = 48 + \frac{2EI}{5} (2\theta_B + 0 - 0) \quad \text{--- (ii)}$$

$$M_{BC} = -80 + \frac{2EI}{5} (2\theta_B + \theta_C - 0) \quad \text{--- (iii)}$$

$$M_{CB} = 80 + \frac{2EI}{5} (2\theta_C + \theta_B - 0) \quad \text{--- (iv)}$$

$$M_{CD} = -25 + \frac{2EI}{5} (2\theta_C + \theta_D - 0) \quad \text{--- (v)}$$

$$M_{DC} = 25 + \frac{2EI}{5} (2\theta_D + \theta_C - 0) \quad \text{--- (vi)}$$

Joint equilibrium,

$$M_{BA} + M_{BC} = 0$$

$$\text{or, } 48 + \frac{2EI}{5} (2\theta_B) - 80 + \frac{2EI}{5} (2\theta_B + \theta_C) = 0$$

$$\text{or, } 1.6\theta_B EI + 0.4\theta_C EI = 32 \quad \text{--- (i)}$$

$$M_{CB} + M_{CD} = 0$$

$$\text{or, } 80 + \frac{2EI}{5} (2\theta_C + \theta_B) - 25 + \frac{2EI}{5} (2\theta_C + \theta_D) = 0$$

$$\text{or, } 0.4\theta_B EI + 1.6\theta_C EI + 0.4\theta_D EI = -55 \quad \text{--- (ii)}$$

$$M_{DC} = 0$$

$$\text{or, } 0.4\theta_C EI + 0.8\theta_D EI = -25 \quad \text{--- (iii)}$$

Solving $\theta_B = \frac{29.711}{EI}$; $\theta_C = \frac{-38.85}{EI}$; $\theta_D = \dots$

Now,

$$M_{AB} = -60.12 \text{ kNm}$$

$$M_{BA} = 71.769 \text{ kNm}$$

$$M_{BC} = -71.769 \text{ kNm}$$

$$M_{CB} = 60.804 \text{ kNm}$$

$$M_{DC} = -60.804 \text{ kNm}$$

$$M_{DC} = 0 \text{ kNm}$$

$$M_{CD} = -25 + \frac{2EI}{34} (2\theta_C + \theta_D - 0) = -25 + \theta_C EI + 0.5\theta_D EI$$

$$M_{DC} = 25 + \frac{2EI}{4} (2\theta_D + \theta_C - 0) = 25 + \theta_D EI + 0.5\theta_C EI$$

Thus,

$$M_{AB} = 0 \Rightarrow \theta_B = \frac{180}{EI}$$

$$M_{BA} + M_{BC} = 0$$

$$\text{or, } 2.1\theta_B EI + 0.25\theta_C EI = 32 \text{ --- (i)}$$

$$M_{CB} + M_{CD} = 0$$

$$\text{or, } 0.25\theta_B EI + 1.5\theta_C EI + 0.5\theta_D EI = -55 \text{ --- (ii)}$$

$$M_{DC} = 0$$

$$\text{or, } 0.5\theta_C EI + \theta_D EI = -25 \text{ --- (iii)}$$

Solving (i), (ii) & (iii)

$$\theta_B = \frac{19.756}{EI}, \theta_C = \frac{-37.951}{EI}; \theta_D = \frac{-6.024}{EI}$$

Thus,

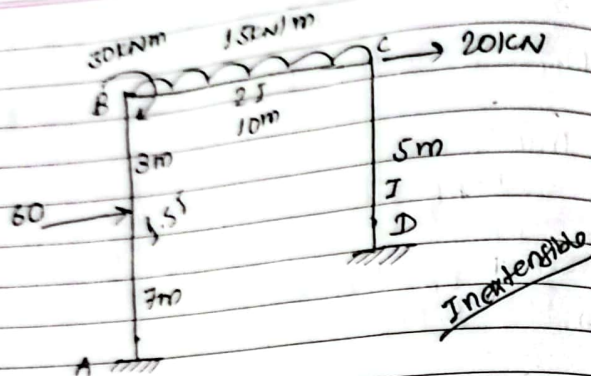
$$M_{AB} = -\frac{64.0976}{EI}; M_{BA} = \frac{79.6096}{EI};$$

$$M_{BC} = -\frac{79.6096}{EI}; M_{CB} = \frac{65.694}{EI};$$

$$M_{CD} = -\frac{65.694}{EI}; M_{DC} = 0.$$

Stiffness Matrix Method

Frame:

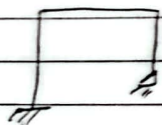


i) DKT

Here,

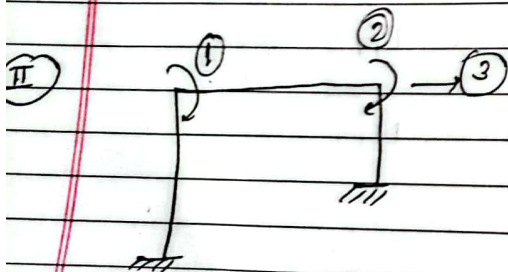
$DSI = 3$ (And $DkI = 3$ as well)

↳ But displacement method will be easier than force method as in force method integrations are to be done (Virtual work method)

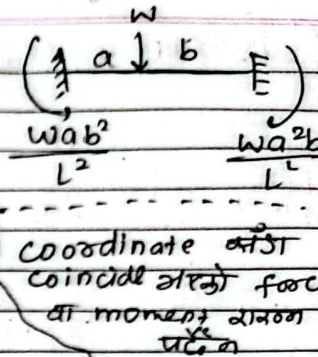
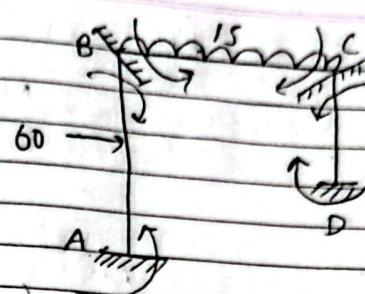


Here, $DkI = 4$

$DSI = 2$ ← easier solving by force method.



FEM

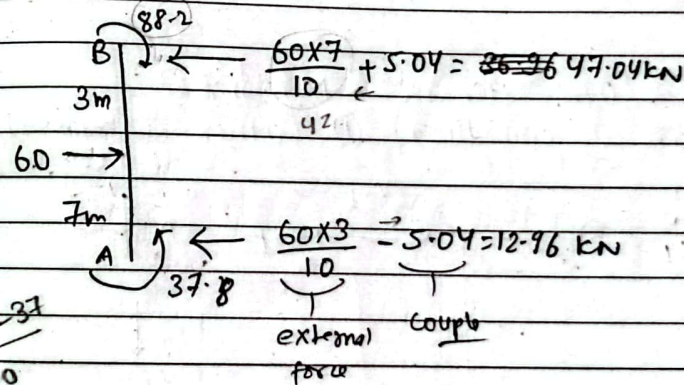


$$FEM_{AB} = -\frac{60 \times 3^2 \times 7}{10^2} = -37.8 \text{ kNm}$$

$$FEM_{BA} = +\frac{60 \times 7^2 \times 3}{10^2} = 88.2 \text{ kNm}$$

$$FEM_{BC} = -\frac{15 \times 10^2}{12} = -125 \text{ kNm}$$

$$FEM_{CB} = 125 \text{ kNm}$$



we only need δ in forces where there is coordinate ①, ② & ③

Inclined load P_1, P_2 coincide $\rightarrow$ line of action of P_1 and P_2 lies in line of action of P_3 so it coincide.

At ① coordinate $P_{1L} = 88.2 - 125 = -36.8 \text{ kNm}$

At ② coordinate $P_{2L} = 125 \text{ kNm}$

At ③ coordinate $P_{3L} = -47.0 \text{ kNm}$

Thus, Net force/moments at coordinate

acting at coordinate $P_1 = 30 \text{ kNm}$
 $P_2 = 0 \text{ kNm}$
 $P_3 = 20 \text{ kNm}$

At ① coordinate $P_{1L} = 88.2 - 125 = -36.8 \text{ kNm}$

At ② coordinate $P_{2L} = 125 \text{ kNm}$

At ③ coordinate $P_{3L} = -47.0 \text{ kNm}$

Thus, Net force/moments at coordinate

acting at coordinate $P_1 = 30 \text{ kNm}$
 $P_2 = 0 \text{ kNm}$
 $P_3 = 20 \text{ kNm}$

Thus, Net force/moments at coordinate

acting at coordinate $P_1 = 30 \text{ kNm}$
 $P_2 = 0 \text{ kNm}$
 $P_3 = 20 \text{ kNm}$

* Generation of stiffness Matrix

• Unit rotation at coordinate ① (other restrained)

Diagram showing unit rotation at coordinate ①. The structure consists of a vertical member of length L and a horizontal member of length L . The vertical member is fixed at the bottom (coordinate ③) and has a unit rotation at the top (coordinate ①). The horizontal member is attached to the top of the vertical member and is fixed at its other end (coordinate ②). The stiffness coefficients are k_{11} , k_{21} , and k_{31} .

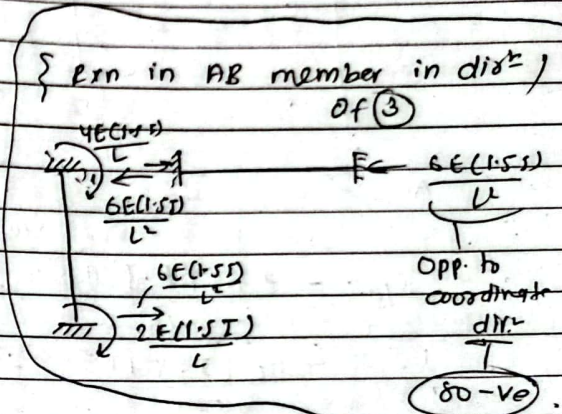
Diagram showing unit rotation at coordinate ①. The structure consists of a vertical member of length L and a horizontal member of length L . The vertical member is fixed at the bottom (coordinate ③) and has a unit rotation at the top (coordinate ①). The horizontal member is attached to the top of the vertical member and is fixed at its other end (coordinate ②). The stiffness coefficients are k_{11} , k_{21} , and k_{31} .

$$k_{11} = \frac{4EI(1.5I)}{10} + \frac{4EI(2I)}{10} = 1.4EI$$

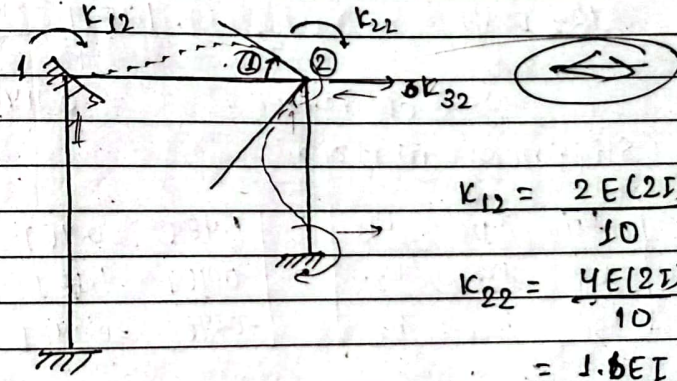
in AB member in BC member

$$k_{21} = \frac{2E(2I)}{10} = 0.4EI$$

$$k_{31} = \frac{-6E(1.5I)}{10E} = -0.9EI$$



• Unit rotation at coordinate ②

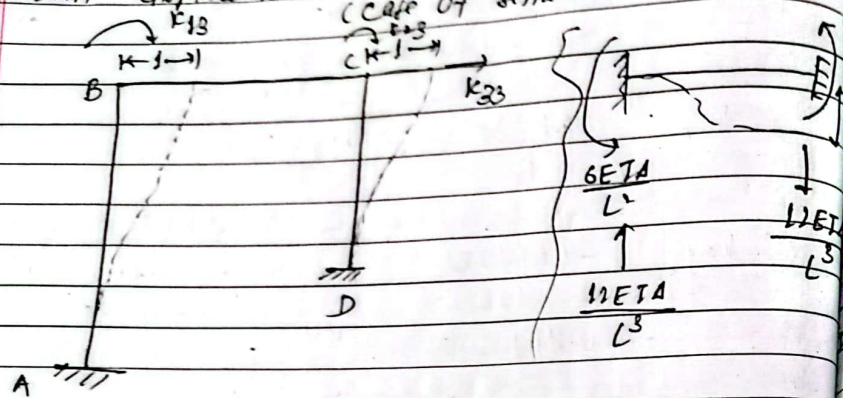


$$k_{12} = \frac{2E(2I)}{10} = 0.4EI$$

$$k_{22} = \frac{4E(2I)}{10} + \frac{4E(I)}{5} = 1.6EI$$

$$k_{32} = -\frac{6E(I)}{5^2} = -0.24EI$$

Unit displacement at ③ along coordinate dir^x (case of settlement)



see AB member
 $k_{13} = -\frac{6E(1.5)(10)}{10^2} = -0.09EI$

$k_{23} = -\frac{6E(1)(10)}{5^2} = -0.24EI$

see CD member
 $k_{33} = \frac{12EI(1)}{5^3} + \frac{12EI(1.5)(11)}{10^3} = 0.114EI$
 see CD member

Now, Stiffness matrix.

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} 1.4EI & 0.4EI & -0.09EI \\ 0.4EI & 1.6EI & -0.24EI \\ -0.09EI & -0.24EI & 0.114EI \end{bmatrix}$$

Equilibrium equations

$$\left. \begin{aligned} k_{11}\Delta_1 + k_{12}\Delta_2 + k_{13}\Delta_3 + P_{1L} &= P_1 \\ k_{21}\Delta_1 + k_{22}\Delta_2 + k_{23}\Delta_3 + P_{2L} &= P_2 \\ k_{31}\Delta_1 + k_{32}\Delta_2 + k_{33}\Delta_3 + P_{3L} &= P_3 \end{aligned} \right\}$$

$\Delta_1, \Delta_2, \Delta_3 \rightarrow$ unknown.

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1.4 & 0.4 & -0.09 \\ 0.4 & 1.6 & -0.24 \\ -0.09 & -0.24 & 0.114 \end{bmatrix}^{-1} \begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} -36.8 \\ 125 \\ -47.04 \end{bmatrix}$$

$\Delta_1 = \theta_B = \frac{90.513}{EI}$

$\Delta_2 = \theta_C = \frac{-2.666}{EI}$

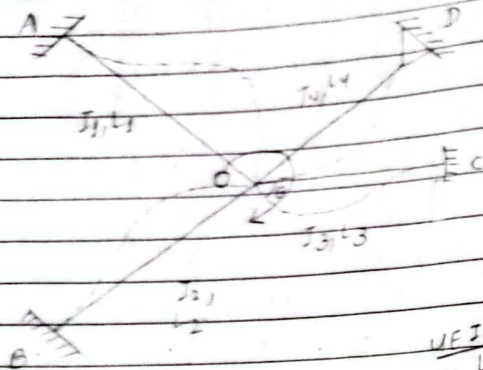
$\Delta_3 = \Delta_{AB} = \Delta_{CD} = \frac{653.915}{EI}$

Now,

Calculate end moments using slope deflection formula:

$$\begin{aligned} M_{AB} &= FEM_{AB} + \frac{2EI}{L} \left(2\theta_A + \theta_B - \frac{3\Delta_{AB}}{L} \right) \\ &= -37.8 + \frac{2E(1.5)}{10} \left(0 + \frac{90.513}{EI} - \frac{3(653.915)}{10EI} \right) \\ &= -69.498 \text{ kNm} \end{aligned}$$

Moment Distribution Method



$$M_{OA} = \frac{4EI_1}{L_1} \theta = K_1 \theta$$

Opposite end fixed

$$M_{OB} = \frac{4EI_2}{L_2} \theta = K_2 \theta$$

$$M_{OC} = \frac{4EI_3}{L_3} \theta = K_3 \theta$$

Opposite end hinged or pinned

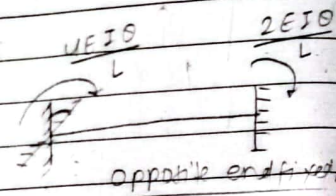
$$M_{OD} = \frac{3EI_4}{L_4} \theta$$

$$= \frac{3}{4} \times \frac{4EI_4}{L_4} \theta$$

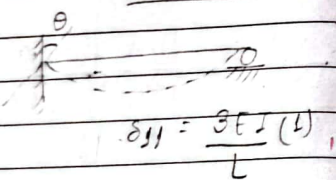
$$= \frac{3}{4} K_4 \theta$$

$$K = \frac{4EI}{L}$$

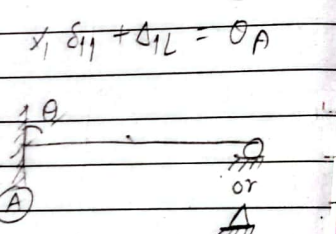
known stiffness
absolute stiffness.



opposite end fixed



$$M_{H} = \frac{3EI}{L} \theta$$



opposite end roller or hinged then

$$M_A = \frac{3EI}{L} \theta$$

At equilibrium,

$$M_{OA} + M_{OB} + M_{OC} + M_{OD} = M$$

$$\text{or, } K_1 \theta + K_2 \theta + K_3 \theta + \frac{3}{4} K_4 \theta = M$$

$$\text{or, } \theta \sum K = M$$

$$\text{or, } \theta = \frac{M}{\sum K}$$

total stiffness

Then,

$$M_{OA} = K_1 \cdot \theta = \frac{K_1}{\sum K} \cdot M$$

* Distribution factor (D)

Joint at which members are unbalanced moment M is to be distributed

Thus,

$$M_{OA} = D_{OA} \times M$$

$$M_{OB} = \frac{K_2}{\sum K} M = D_{OB} M$$

$$D.F. = \frac{K}{\sum K} = \frac{\frac{4EI}{L}}{\sum \frac{4EI}{L}}$$

$\frac{I}{L} \rightarrow$ Relative stiffness.

Distribution factor

$$(D.F.) = \frac{K_x}{\sum K}$$

(4E) common for all

So,

Just take $\frac{I}{L}$

Relative stiffness

$$\frac{4EI}{L} \rightarrow \text{absolute stiffness}$$

$$M_{oc} = \frac{k_3}{\sum k} M = D_{oc} M$$

$$M_{od} = \frac{k_4}{\sum k} M = D_{od} M$$

$$\frac{5 \times 4 \times E I}{L}$$

Carry-over factor

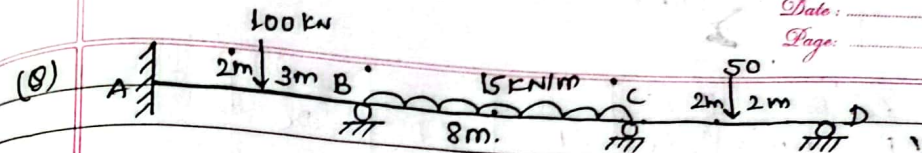
$$\frac{M_{BA}}{M_{AB}} = \frac{\left(\frac{2EI\theta}{L}\right)}{\left(\frac{4EI\theta}{L}\right)} = \frac{1}{2}$$

Carry-over factor } $\frac{1}{2}$ set support rotation θ at
 } θ at support A and B moment
 } θ at support A and B rotation

need to multiply
 by this factor with moment at ~~set~~ rotated support
 to get moment at opposite support

$$M_{AB} = \frac{4EI\theta}{L}$$

$$M_{BA} = \frac{2EI\theta}{L}$$



(Q) Soln:

• Joint B (B and C)

Distribution factor

$$D_{BA} = \frac{\left(\frac{I}{L}\right)}{\sum \left(\frac{I}{L}\right) \leftarrow \text{Joint B only}} = \frac{\left(\frac{I}{5}\right)}{\left(\frac{I}{5}\right) + \left(\frac{I}{8}\right)} = \frac{8}{13}$$

$$D_{BC} = \frac{\left(\frac{I}{8}\right)}{\left(\frac{I}{5}\right) + \left(\frac{I}{8}\right)} = \frac{5}{13}$$

• Joint C

$$D_{CB} = \frac{\frac{I}{8}}{\frac{I}{8} + \frac{3 \times \frac{I}{4}}{2}} = \frac{4}{14} = \frac{2}{7}$$

end at roller support

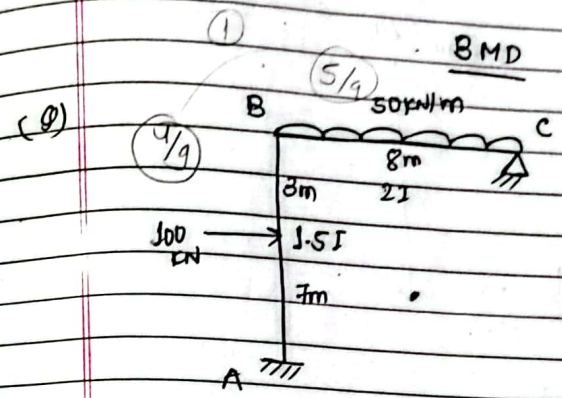
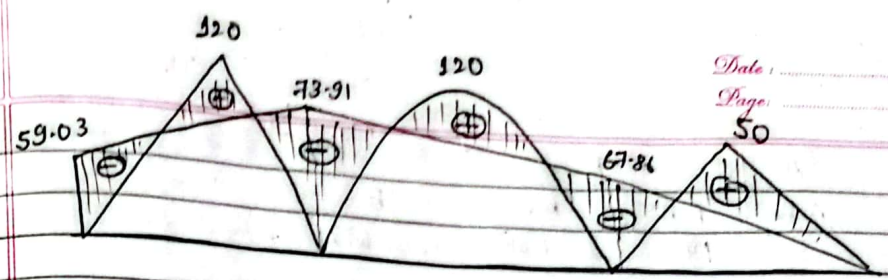
$$D_{CD} = \frac{\frac{3}{4} \times \frac{I}{4}}{\frac{I}{8} + \frac{3}{4} \times \frac{I}{4}} = \frac{3}{5}$$

hill 0.0 -32 → unbalance d moment at joint B

Joint	A	B	C	D
Member	AB	BA BC	CB CD	DC
D.F.	-	5/13 5/13	2/5 3/5	-
F.E.M.	-72	+48 -80	+80 -25	+25
Balance	-	+19.69 +12.31	-22 -33	-25
Carry over	+9.845	-	6.155 -12.5	-
Balance	-	+6.77 +4.23	+2.58 +3.81	-
Carry over	3.385	-	1.27 2.115	-
Balance	-	-0.78 -0.488	-0.846 -1.269	-
Carry over	-0.39	-	-0.423 -0.244	-
Balance	-	+0.26 +0.163	+0.098 +0.146	-
Carry over	0.13	-	0.049 0.0815	-
Balance	-	-0.030 -0.019	-0.0326 -0.0489	-

Now, FEM को distribute करे आरको सह जोडे

Final	-72				
nd	9.845				
ment	3.385				
	-0.39	73.91	67.865	-67.862	0
	+0.13	-73.91			
	-59.03				



→ Solution,
Distribute only at B

$$D_{BA} = \frac{\left(\frac{1.57}{10}\right)}{\left(\frac{1.57}{10}\right) + \frac{3}{4} \times \frac{27}{8}} = \frac{4}{9}$$

$$D_{BC} = \frac{\frac{3}{4} \times \frac{27}{8}}{\frac{1.57}{10} + \frac{3}{4} \times \frac{27}{8}} = \frac{5}{9}$$

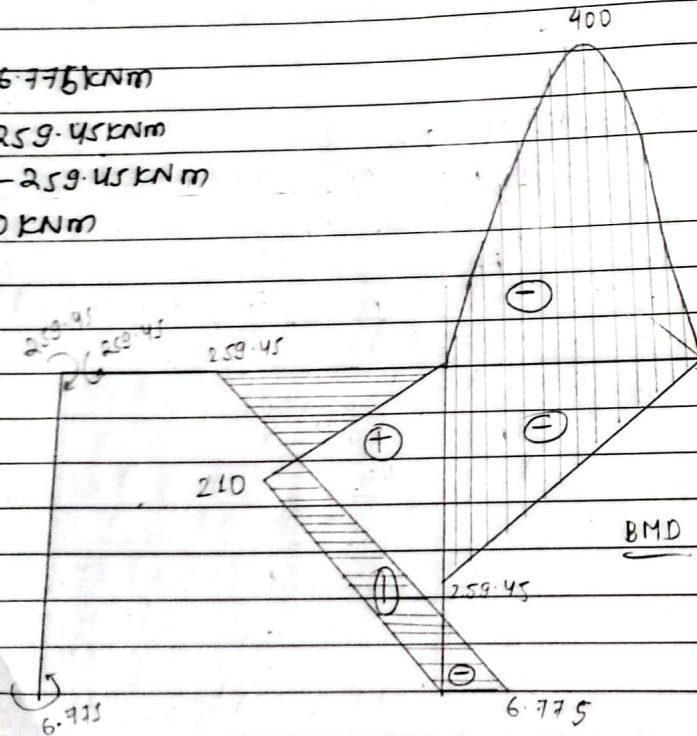
Joint	A		B	C
Member	AB	BA	BC	CB
FEM	-63	147	-266.667	+266.677
D.F	-	$\frac{4}{9}$	$\frac{5}{9}$	-
Balance	-	+53.19	+66.48	-266.677
C.O	26.595	-	-133.334	33.24
Balance	-	+53.260	74.07	-
C.O	29.63	-	-	-
Final Balance	-6.775	259.45	-259.45	0

$$M_{AB} = -6.775 \text{ kNm}$$

$$M_{BA} = 259.45 \text{ kNm}$$

$$M_{BC} = -259.45 \text{ kNm}$$

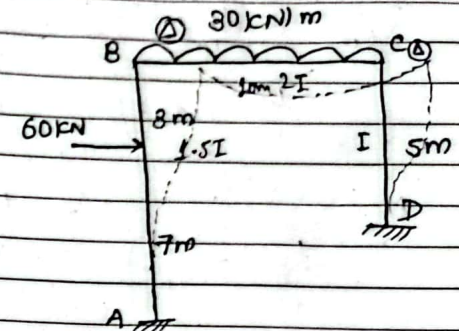
$$M_{CB} = 0 \text{ kNm}$$



Frame with side sway

→ Soln:-

Moment distribution
w/o side sway



$$FEM_{AB} = \frac{-60 \times 3^2 \times 7}{10^2} = -37.8 \text{ kNm}$$

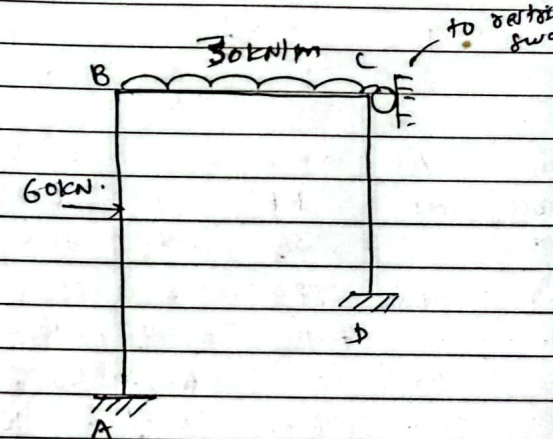
$$FEM_{BA} = \frac{+60 \times 7^2 \times 3}{10^2} = 88.2 \text{ kNm}$$

$$FEM_{BC} = \frac{-30 \times 10^2}{12} = -250 \text{ kNm}$$

$$FEM_{CB} = 250 \text{ kNm}$$

$$FEM_{CD} = 0$$

$$FEM_{DC} = 0$$



Distribution factor,

$$D_{BA} = \frac{\left(\frac{1.5I}{10}\right)}{\left(\frac{1.5I}{10}\right) + \left(\frac{2I}{10}\right)} = \frac{3}{7}$$

$$D_{BC} = \frac{\left(\frac{2I}{10}\right)}{\left(\frac{1.5I}{10}\right) + \left(\frac{2I}{10}\right)} = \frac{4}{7}$$

$$D_{CB} = \frac{\left(\frac{2I}{10}\right)}{\left(\frac{2I}{10}\right) + \left(\frac{I}{5}\right)} = 0.5$$

$$D_{CD} = \frac{\left(\frac{I}{5}\right)}{\left(\frac{2I}{10}\right) + \left(\frac{I}{5}\right)} = 0.5$$

Joint	A	B	C	D		
Members	AB	BA	BC	CB	CD	DC
D.F.	-	$\frac{3}{7}$	$\frac{4}{7}$	0.5	0.5	-
FEM	-37.8	22.2	-250	250	0	0
Bal.	-	+69.34	+92.46	-125	-125	-
CO.	34.67	-	-62.5	46.23	-	-62.5
Bal.	-	+26.78	+35.71	-23.115	-23.115	-
CO.	13.39	-	-11.56	18.86	-	-11.56
Bal.	-	+4.95	+6.61	-9.43	-9.43	-
CO.	2.475	-	-4.715	3.305	-	-4.715
Bal.	-	+2.021	+2.694	-1.652	-1.652	-
CO.	1.0105	-	-0.826	1.347	-	-0.826
Bal.	-	+0.354	+0.472	-0.673	-0.673	-
CO.	0.177	-	-0.336	0.236	-	-0.336

		+0.144	+0.192	0.118	0.118	-
CO	0.072	-	+0.059	+0.036	-	+0.059
Bal.	-	-0.025	-0.034	-0.048	-0.048	-

2.

$$M_{AB} = 13.994 \text{ kNm}$$

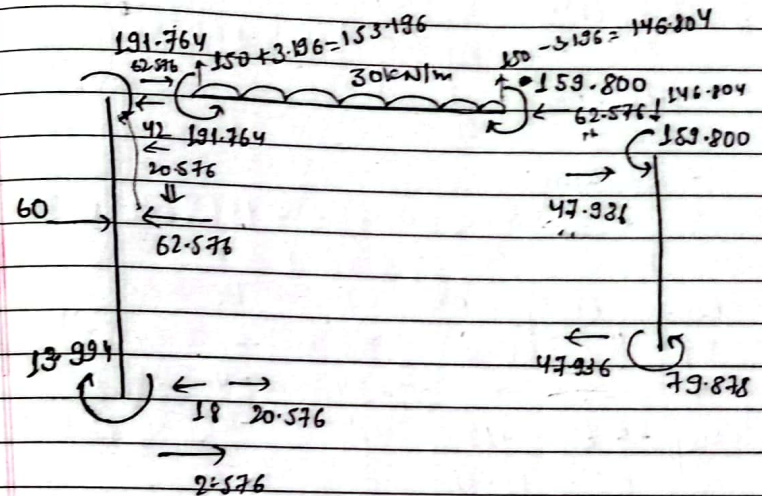
$$M_{BA} = 191.764 \text{ kNm}$$

$$M_{BC} = -191.774 \text{ kNm}$$

$$M_{CB} = 160.878 \text{ kNm}$$

$$M_{CD} = -159.800 \text{ kNm}$$

$$M_{DC} = -79.878 \text{ kNm}$$



$$\sum F_x = 0 \quad (\rightarrow +ve) \quad \text{A.C}$$

$$2.576 - 47.973 + 60$$

$$F_x = 62.576 - 47.936$$

$$[X = 14.64 \text{ kN}] (\rightarrow)$$

Sway calculation.

Let $EI\Delta = 100$

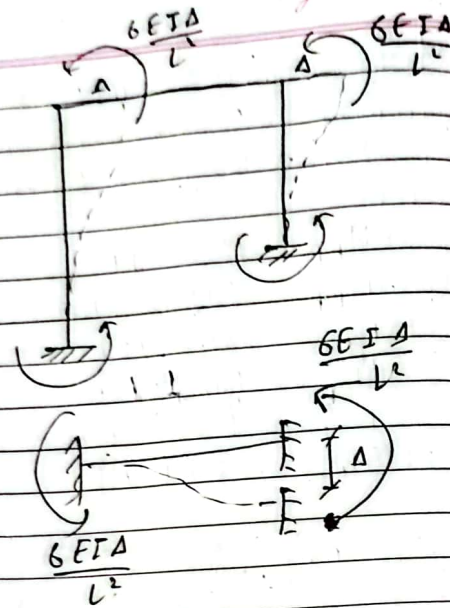
$$FEM_{AB} = -\frac{6E(1.5)\Delta}{L^2}$$

$$= -9 \text{ kNm}$$

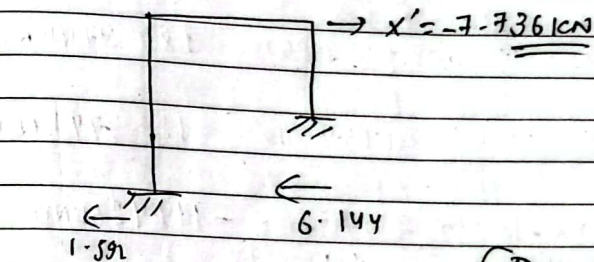
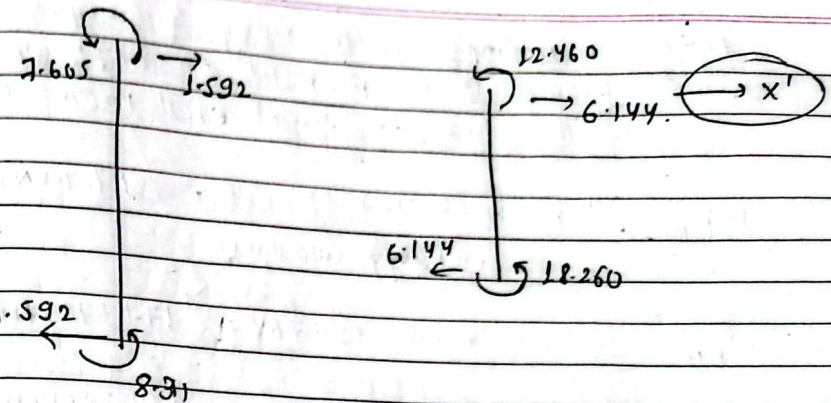
$$FEM_{CD} = -\frac{6E(7)\Delta}{L^2}$$

$$= -\frac{6E(7)\Delta}{5^2}$$

$$= -24 \text{ kNm}$$



Joint	A	B	C	D		
Member	AB	BA	BC	CB	CD	DC
DF	-	3/7	4/7	0.5	0.5	-
FEM	-9	-9	-	-	-24	-24
Bal	-	+3.857	+5.143	+12	+12	-
CO	-1.928	-	6	-2.571	-	6
Bal	-	-2.571	-3.429	-1.285	-1.285	-
CO	-1.285	-	-0.642	-1.714	-	-0.642
Bal	-	+0.275	+0.357	+0.857	+0.857	-
CO	+0.137	-	+0.428	+0.183	-	+0.428
Bal	-	-0.183	-0.244	-0.091	-0.091	-
CO	-0.091	-	-0.046	-0.122	-	-0.046
Bal	-	+0.020	+0.026	+0.061	+0.061	-
Σ	-8.311	-7.605	+7.605	+12.460	-12.460	-18.260



(X) → restraining force required s.t. no sway occurs.

$$X - \lambda X' = 0$$

or, $\lambda = \frac{X}{X'}$ ← w/o considering sway

$X' \leftarrow$ assuming sway

$$= \frac{12.460}{7.736}$$

$$= 1.892$$

→ Multiplying factor (+ve)

→ Correction factor

1.892 multiply with sway moments and add ~~more~~ it to moment in restrained condition to get final end moment.

$$M_{AB} = 1.892 \times -8.311 + 19.994 = -1.730 \text{ kNm}$$

with sway w/o sway

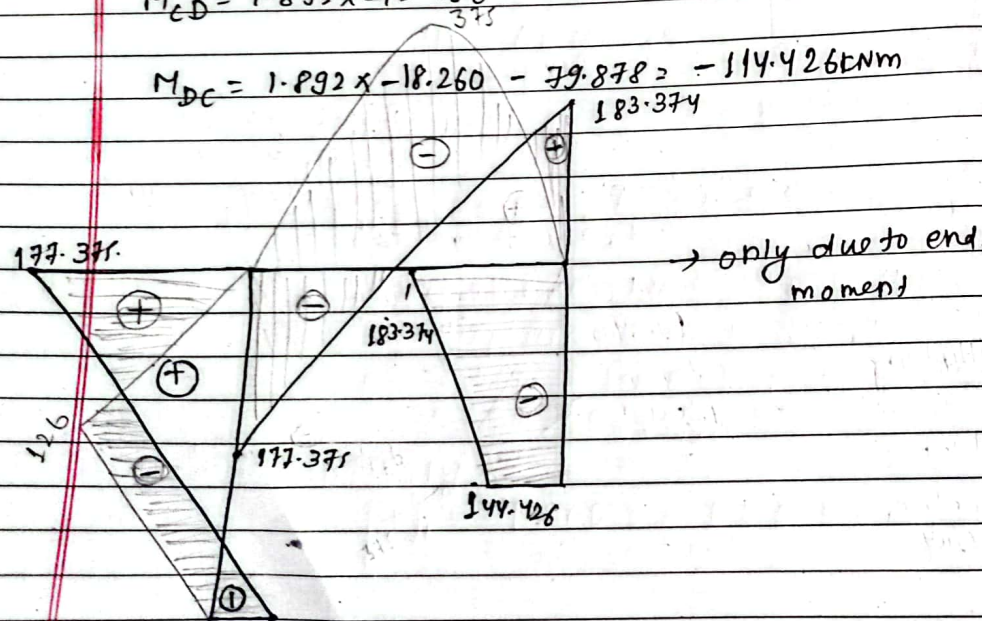
$$M_{BA} = 1.892 \times -7.605 + 191.764 = 177.375 \text{ kNm}$$

$$M_{BC} = 1.892 \times +7.605 - 191.764 = -177.375 \text{ kNm}$$

$$M_{CB} = 1.892 \times 12.460 + 159.800 = 183.374 \text{ kNm}$$

$$M_{CD} = 1.892 \times -12.460 - 159.800 = -183.374 \text{ kNm}$$

$$M_{DC} = 1.892 \times -18.260 - 79.878 = -114.426 \text{ kNm}$$



Case of Symmetry and Anti-symmetry:

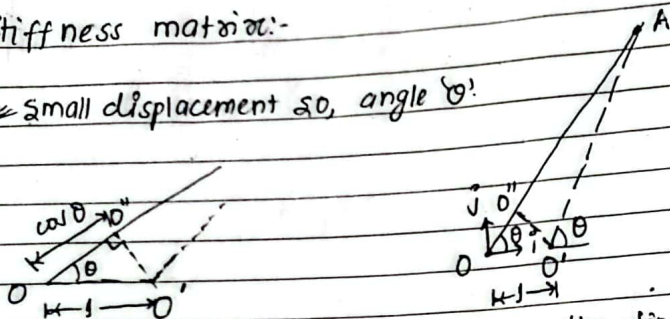
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TRUSS: Displacement Method

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Stiffness matrix:-

Small displacement so, angle θ'



- Unit displacement along i-direction:
OA member shorten by $\cos \theta$ when O reaches O'

$$\Delta L = \frac{PL}{AE}$$

$$\cos \theta = \frac{PL}{AE}$$

$$\Rightarrow [P = \frac{AE \cos \theta}{L}]$$

P $\rightarrow$ Axial force in member

Now,

$$K_{ii} = P \cos \theta = \sum \frac{AE \cos^2 \theta}{L}$$

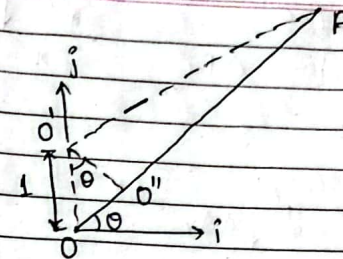
force at i coordinate due to unit displacement in 'i' direction

$\theta' \rightarrow$ ACW angle from +ve x-axis

Similarly,

$$[K_{ji} = P \sin \theta = \sum \frac{AE \cos \theta \sin \theta}{L}]$$

Date: _____
Page: _____



Here, unit displacement along j-direction

$$OO' = \sin \theta$$

Then,

$$\Delta L = \frac{PL}{AE} = \sin \theta$$

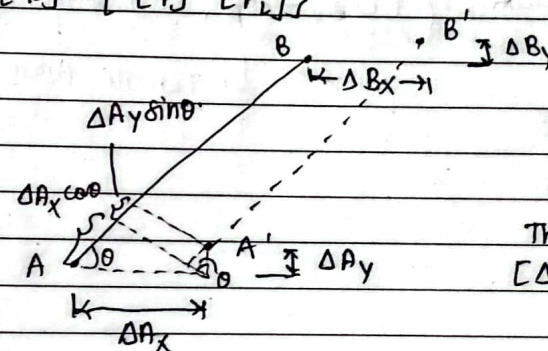
$$[P = \frac{AE \sin \theta}{L}]$$

Now,

$$[K_{ij} = P \cos \theta = \sum \frac{AE \sin \theta \cos \theta}{L}]$$

$$[K_{jj} = P \sin \theta = \sum \frac{AE \sin^2 \theta}{L}]$$

$$[\Delta] = [K]^{-1} \{ [P] - [P_L] \}$$

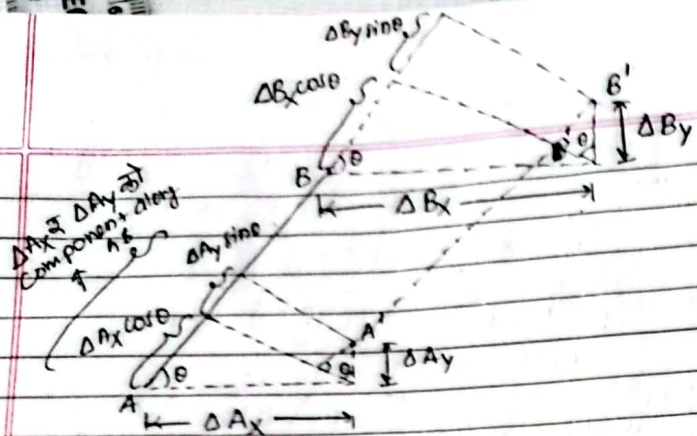


Here,

$$\{ [P_L] = 0 \}$$

Thus,

$$[\Delta] = [K]^{-1} [P]$$



ΔA_x & ΔA_y को component ले $\rightarrow$ AB लाई shorten गर्छ

ΔB_x & ΔB_y को component ले $\rightarrow$ AB लाई elongate गर्छ

Now,

Net change in length of AB

(OR Net ~~red~~ ^{elongation}), ΔL

$$\Delta L = +\Delta B_x \cos \theta + \Delta B_y \sin \theta - \Delta A_x \cos \theta - \Delta A_y \sin \theta$$

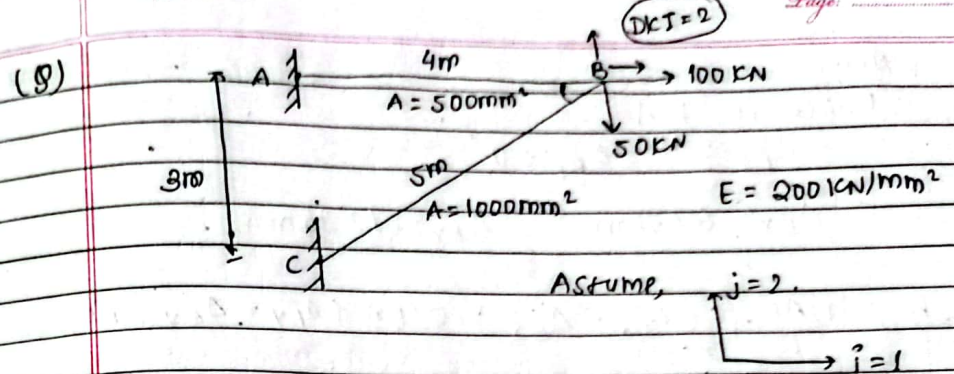
$$\Delta L = f(\Delta B_x - \Delta A_x) \cos \theta + (\Delta B_y - \Delta A_y) \sin \theta$$

$$\frac{PL}{AE} = f(\Delta B_x - \Delta A_x) \cos \theta + (\Delta B_y - \Delta A_y) \sin \theta$$

$$P = \frac{f}{L} AE [(\Delta B_x - \Delta A_x) \cos \theta + (\Delta B_y - \Delta A_y) \sin \theta]$$

$\hookrightarrow$ Tensile force

Determine force in members using displacement method.



$\rightarrow$ Solution,

Member	$\frac{AE}{L}$ (kN/mm)	θ	$\cos \theta$	$\sin \theta$	$\frac{AE \cos^2 \theta}{L}$	$\frac{AE \cos \theta \sin \theta}{L}$	$\frac{AE \sin^2 \theta}{L}$
BA	25	180°	-1	0	25	0	0
BC	40	216.8°	$-\frac{4}{5}$	$-\frac{3}{5}$	25.6	19.2	14.4
Σ					50.6	19.2	14.4

$$K_{11} = \frac{\Sigma AE \cos^2 \theta}{L} = 50.6 \text{ kN/mm}$$

$$K_{12} = K_{21} = \frac{\Sigma AE \cos \theta \sin \theta}{L} = 19.2 \text{ kN/mm}$$

$$K_{22} = \frac{\Sigma AE \sin^2 \theta}{L} = 14.4 \text{ kN/mm}$$

$$[D] = [K]^{-1} [P]$$

$$= \begin{bmatrix} 50.6 & 19.2 \\ 19.2 & 14.4 \end{bmatrix}^{-1} \begin{bmatrix} -50 & 100 \\ -50 \end{bmatrix}$$

$$\begin{Bmatrix} \Delta_i \\ \Delta_j \end{Bmatrix} = \begin{bmatrix} 6.667 \\ -12.361 \end{bmatrix} \text{ mm}$$

Now

Force in BA

$$\Delta_{Ax} = 0 \quad \Delta_{Ay} = 0$$

$$\Delta_{Bx} = 6.67 \text{ mm} \quad \Delta_{By} = -12.36 \text{ mm}$$

$$P_{BA} = \frac{EA}{L} [(\Delta_{Ax} - \Delta_{Bx}) \cos \theta + (\Delta_{Ay} - \Delta_{By}) \sin \theta]$$

$= \frac{500 \times 500 \times 200}{4000} \cdot \left(\begin{array}{l} \text{नजिकको छाप/टाढाको छाप/टाढाको} \\ \text{B मा बसेको छाप/टाढाको} \\ \text{(टाढाको छाप नजिकको छाप/टाढाको)} \\ \text{(B मा बसेको छाप/टाढाको)} \end{array} \right)$

$$= \frac{500 \times 200}{4000} [(0 - 6.67)(-1) + (0 + 12.36)(0)]$$

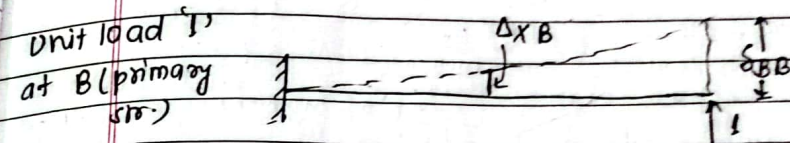
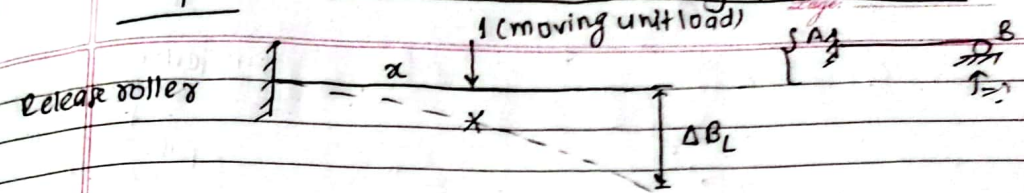
$$= 166.75 \text{ kN (T)}$$

$$P_{BC} = \frac{EA}{L} [(\Delta_{Cx} - \Delta_{Bx}) \cos \theta + (\Delta_{Cy} - \Delta_{By}) \sin \theta]$$

$$= \frac{1000 \times 200}{5000} [(0 - 6.67)(-0.8) + (0 + 12.36)(-0.6)]$$

$$= -83.20 \text{ kN}$$

$$= 83.20 \text{ kN (C)}$$



$$\Delta_{BL} = \Delta_{XB} \text{ (Maxwell's reciprocal theorem)}$$

compatibility eqn,

$$\Delta_B = 0$$

$$\text{or, } R_{By} \cdot \delta_{BB} - \Delta_{BL} = 0$$

$$\left[R_{By} = \frac{\Delta_{BL}}{\delta_{BB}} = \frac{\Delta_{XB}}{\delta_{BB}} \right]_{\text{u}}$$

displacement value when unit load at x

δ_{BB} (constant wherever unit load)

→ Muller Breslau Principle.

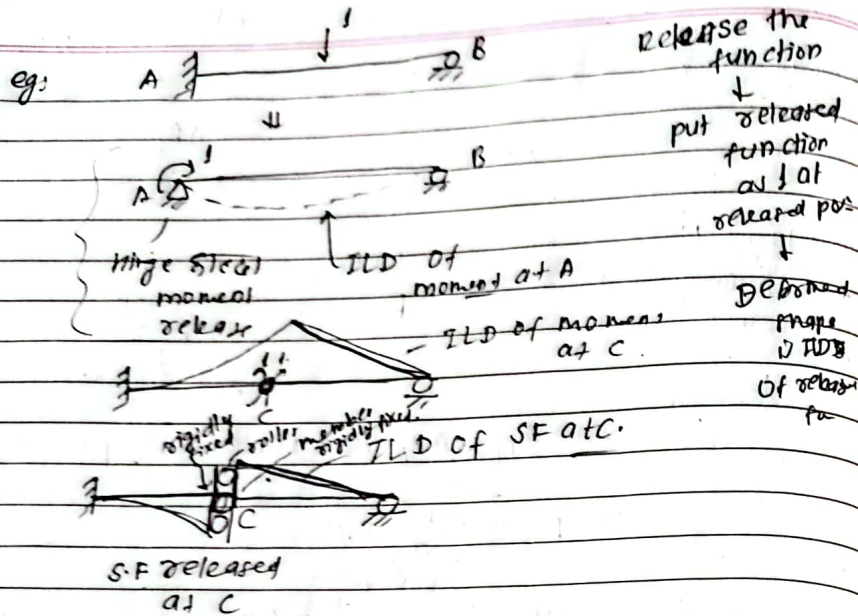
• Muller Breslau Principle:

If a function in a structure is released and a value of ^{that} function is given to the structure the resulting deformed shape of the structure will be the shape of the influence line diagram of that function.

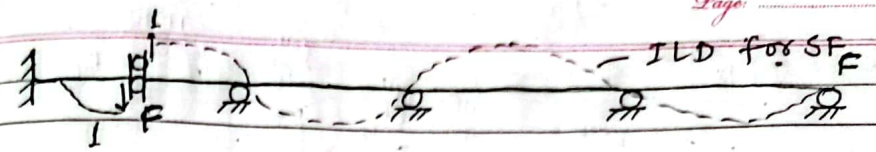
{ Above, functⁿ → R_{By}

released to form resulting deformed shape which is shape of ILD of R_{By} .

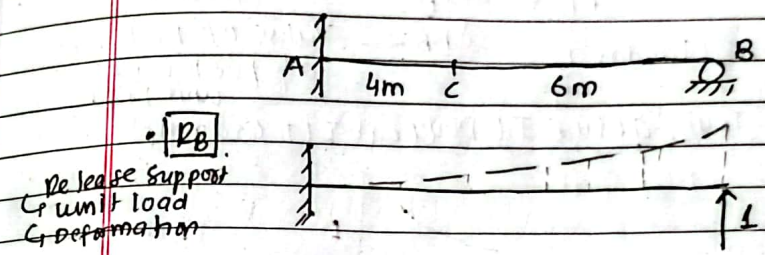
Indeterminate $\rightarrow$ ILD always curve



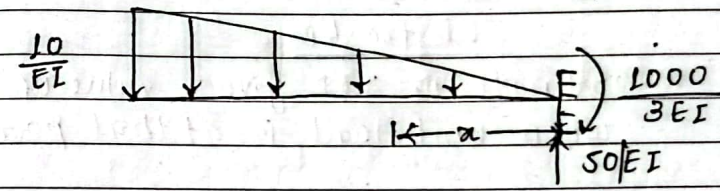
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ILD for propped cantilever
 $R_B, R_A, M_A, (SF)_C, (BM)_C$



Use conjugate beam method to find deflection at interval of 2m.



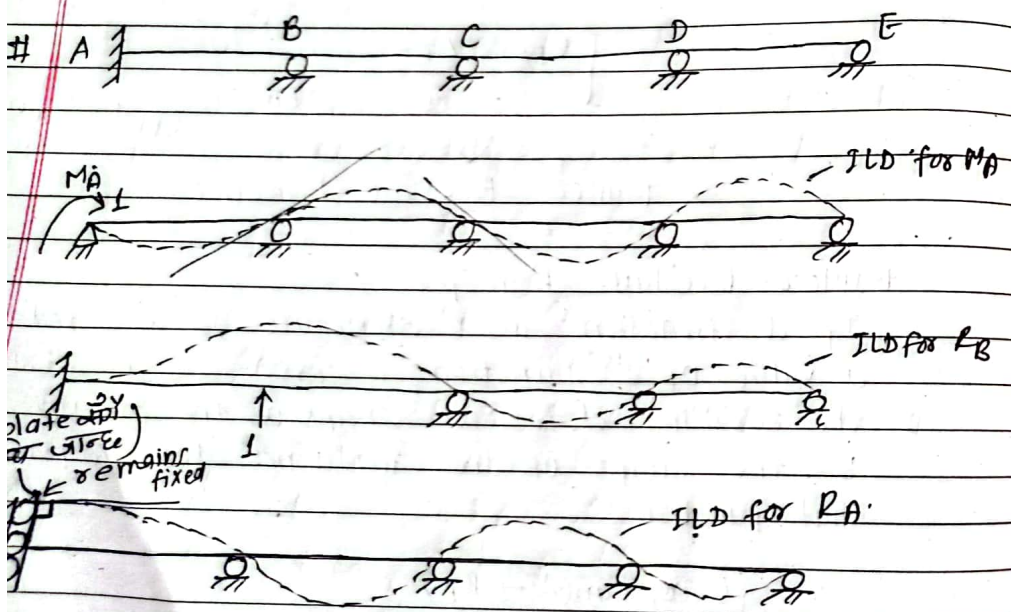
Write equation in terms of x to find BM at each interval

$$M_a = \frac{50x}{EI} - \frac{1000}{3EI} - \frac{1}{2} \times \left(\frac{x}{EI} \right) \times x \times \frac{x}{3}$$

$$= \frac{50x}{EI} - \frac{1000}{3EI} - \frac{x^3}{6EI}$$

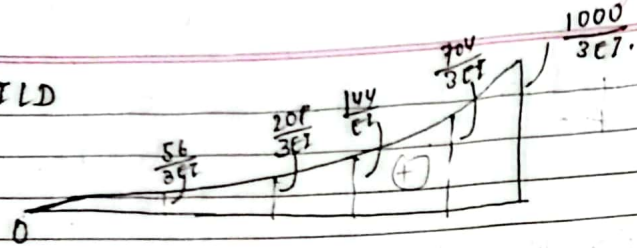
At 2m = $-\frac{704}{3EI}$
 4m = $-144/EI$
 6m = $-208/3EI$
 8m = $-56/3EI$
 10m = 0

$\left\{ \begin{array}{l} (BM)_{conjugate\ beam} = (\Delta)_{real\ beam} \end{array} \right\}$



not critical
 i.e. (only release R_A not M_A) $\rightarrow$ fixed support & so moment release of M_A

ILD



But, for ILD,

Δ_{XB}

$R_B =$

Δ_{XB}

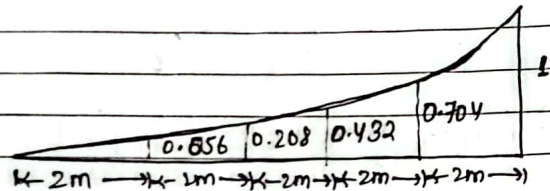
Deflection at any position

Defⁿ at X unit load at B

δ_{BB}

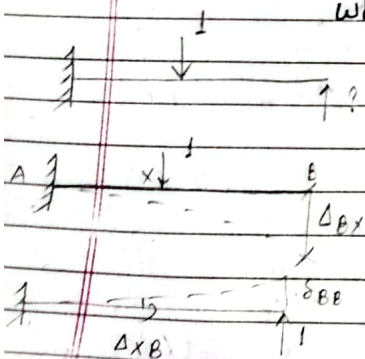
value of Deflection at position of unit load

Thus, actual ILD (Real ILD ordinate)



ILD for R_B

ordinate of ILD gives value of R_B when unit load is at that position.



$$R_B \cdot \delta_{BB} - \Delta_{BX} = 0$$

$$R_B = \frac{\Delta_{BX}}{\delta_{BB}}$$

From Maxwell's reciprocal principle

$$\Delta_{BX} = \Delta_{XB}$$

$$\left\{ \begin{array}{l} R_B = \frac{\Delta_{XB}}{\delta_{BB}} \end{array} \right\}$$

$$R_A + R_B = 1$$

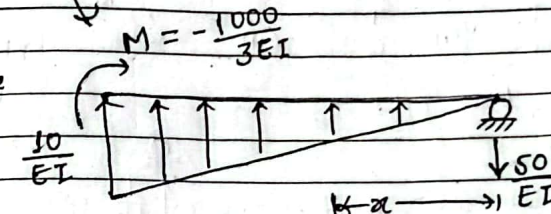
R_A

Release R_A (by roller)

Put unit load at A

Deflection shape analyse

M
↓
Beam stable
dashed



Conjugate beam.

Deflection shape

(need to find deflection at each location)

BMD arrow toward compression side

A is still fixed support

M moment is kept at released position

(s.t. free end) zero moment

$$M_x = -\frac{50}{EI}x + \frac{1}{2} \times \left(\frac{x}{EI} \right) \times x \times \frac{x}{3}$$

Stable beam develop at M. balancing M

$$\begin{array}{l} \text{At } 2m = -\frac{296}{3EI} \\ 4m = -\frac{568}{3EI} \\ 6m = -\frac{264}{EI} \\ 8m = -\frac{844}{3EI} \\ 10m = -\frac{1000}{3EI} \end{array}$$

Why M at free end?

जो त stable है

अनि conjugate

beam को rxn ले जग

moment नि develop

होगा? हा, जो moment

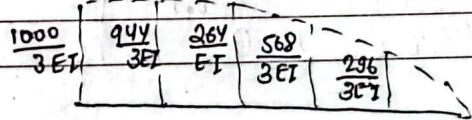
कहा जावगे?

$(M)_{conj} = (\text{deflect})_{real}$

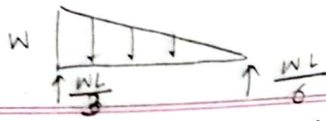
↓ deflection A न

भरको है 50, M

keep at A



Note:



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ILD coordinate

ILD for R_A is:

ILD of R_A

ILD for R_B

unit load

deflection

Here deflection

i.e. momental

that part of

conjugate beam

4th 1000

3EI

Release moment

at A

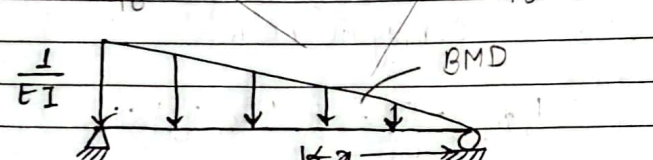
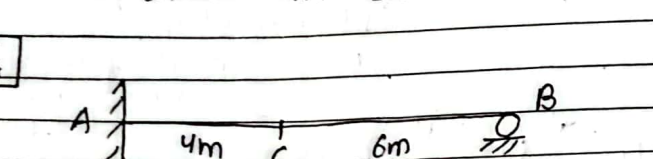
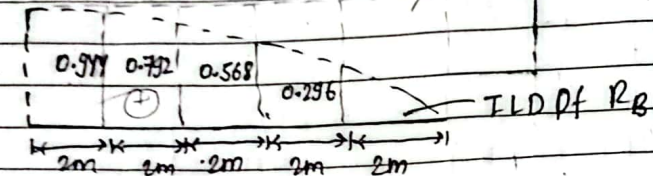
(By hinge

support at

A)

unit moment

at A



At a,

$$M_a = \frac{5}{3EI} a - \frac{1}{2} \times \left(\frac{a}{EI} \right) \frac{a \times a}{3} + \frac{1}{10} a$$

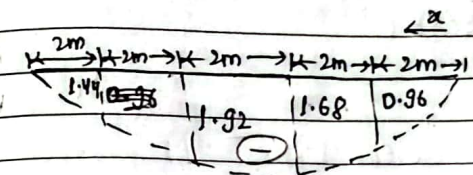
At 2m =	$\frac{2}{EI}$	$\frac{16}{5EI}$
4m =	$\frac{4}{EI}$	$\frac{28}{5EI}$
6m =	$-\frac{26}{EI}$	$\frac{32}{5EI}$
8m =	$-\frac{72}{EI}$	$\frac{24}{5EI}$
10m =	$-\frac{150}{EI}$	0

δ_{11} → rotation at A → i.e. SF at A in conjugate beam
unit rotation at A slope develop

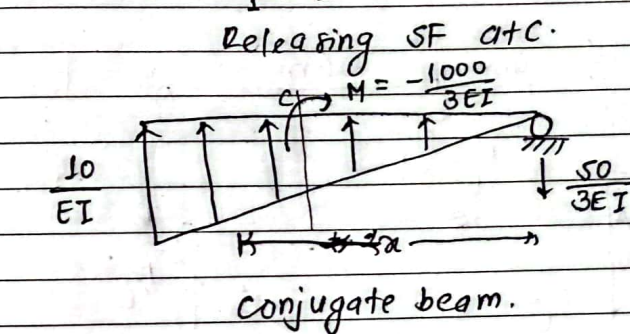
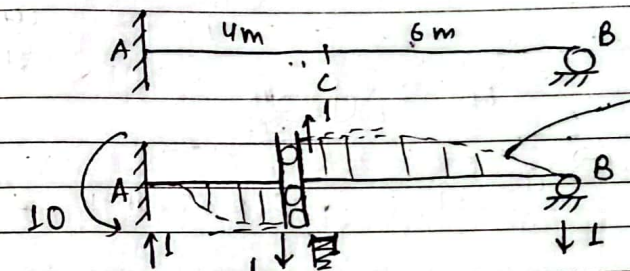
Here, $\delta_{11} = \frac{10}{3EI}$

$M_A = \frac{\Delta \times P}{\delta_{11}}$

ILD of M_A



(SF)_c



Due to M_A ,
 δ_{11} is developed
at released position
(δ_{11}) → slope

(SF) conjugate
at A

$\frac{10}{3EI}$

Divide by this
quantity

deflection
shape due
to displacement
in real beam
= moment
in conj
beam
• 'c' position
it is seen
jump in displacements
there must
be couple at
that position
in conjugate
beam

(0-6 6-10) ← Diff. Page

$$M_D = -\frac{50a}{EI} + \frac{1}{2} \frac{a \cdot a \cdot a}{EI \cdot 3} + \frac{1000}{3EI} (a-6)^0$$

$$At \ a=2m = -296/3EI$$

$$a=4m = -568/3EI$$

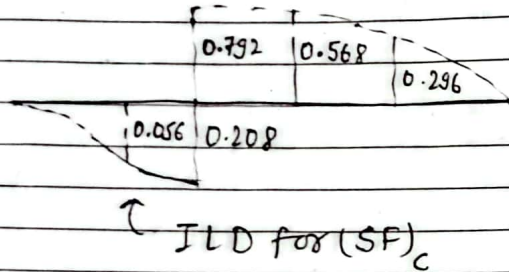
$$(a=6m)_R = -264/3EI$$

$$(a=6m)_L = 208/3EI$$

$$a=8m = 56/3EI$$

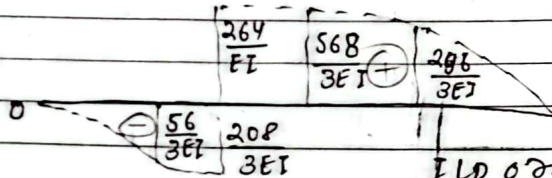
$$a=10m = 0$$

(SF at divide)
M at divide



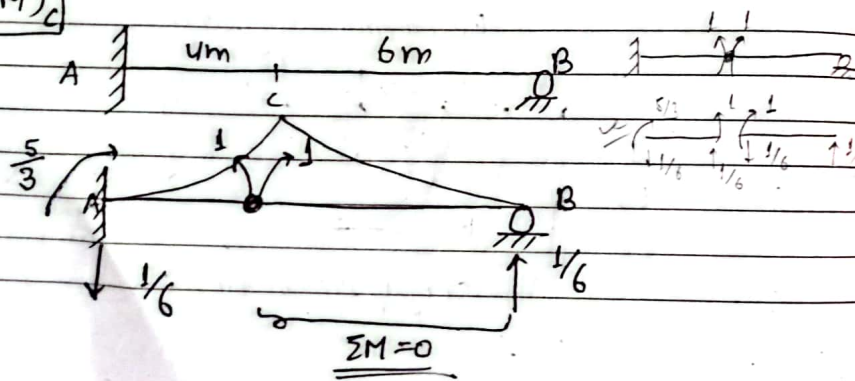
$$(SF)_C = \frac{\Delta_{DB}}{M}$$

Unit load
Rt of C
(SF)_C = R_A
Unit load
Left of C
(SF)_C = R_B



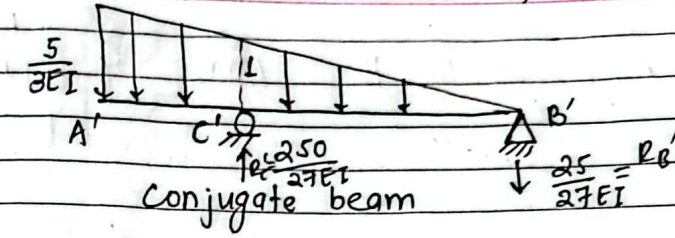
$$ILD \ at \ divide = \frac{\Delta}{M_C'}$$

(BM)_C



$$\Sigma M = 0$$

Trick: Divide? → Force at divide moment at divide
Moment at divide Force at divide
unit load/released position at divide force moment



$$M_D = -\frac{25}{27EI} a + \frac{250}{27EI} (a-6)^1 - \frac{1}{2} \times \left(\frac{5}{3EI}\right) \times \left(\frac{a}{10}\right) \times \frac{a^2}{3}$$

$$0m = 0$$

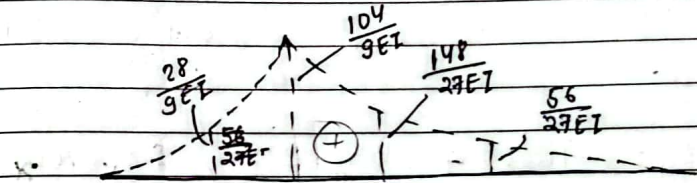
$$At \ 2m = -106/27EI - 56/27EI$$

$$4m = -448/27EI - 148/27EI$$

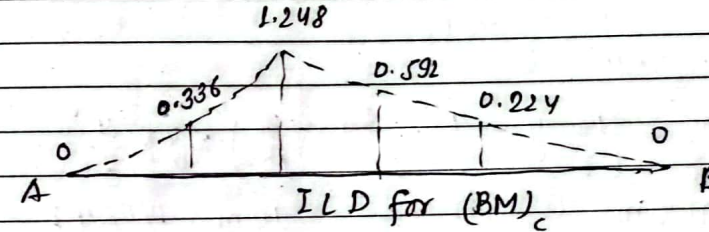
$$6m = -118/3EI - 104/9EI$$

$$8m = -1484/27EI - 28/9EI$$

$$10m = -250/3EI - 0$$

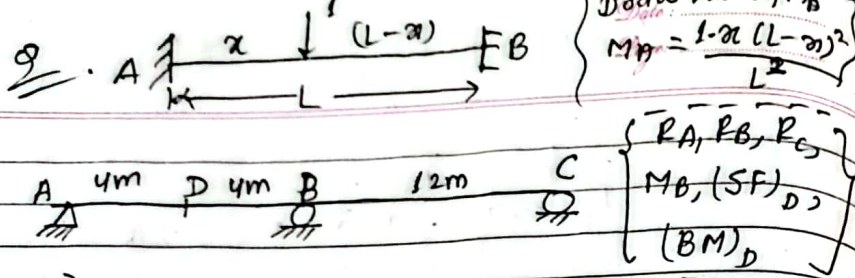


$$(BM)_C = \frac{ILD \ at \ divide \ by \ R_C' = \frac{250}{27EI}}{R_C'}$$

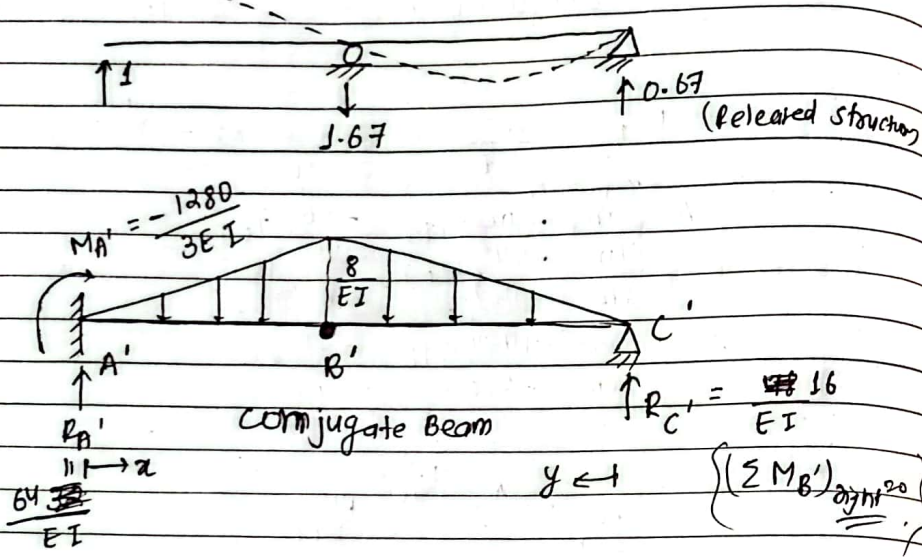


$$ILD \ at \ divide = \frac{\Delta}{R_C'}$$

Divide by M_B
(9)



can't release by keeping vertical roller.



$$M_x = -\frac{1280}{3EI} + \frac{64}{EI}x - \frac{1}{2} \times \left(\frac{8}{EI}\right) \times \left(\frac{x}{8}\right) \times \frac{x}{3}$$

$$0m = -\frac{1280}{3EI}$$

$$At 2m = -\frac{300}{EI}$$

$$4m = -\frac{544}{3EI}$$

$$6m = -\frac{236}{3EI}$$

$$8m = 0$$

$$M_y = \frac{16}{EI}y - \frac{1}{2} \times \left(\frac{8}{EI}\right) \times \left(\frac{y}{12}\right) \times y \times \frac{y}{3}$$

$$At 0m = 0$$

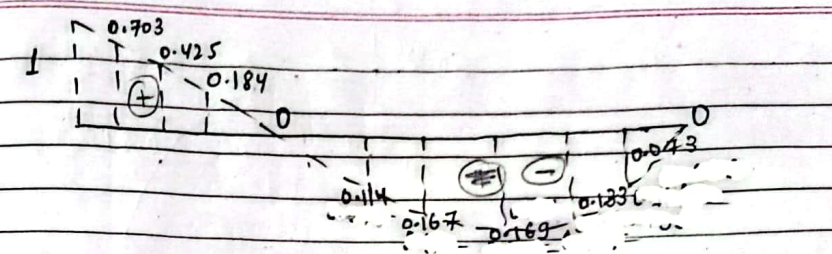
$$2m = \frac{280}{9EI}$$

$$4m = \frac{512}{9EI}$$

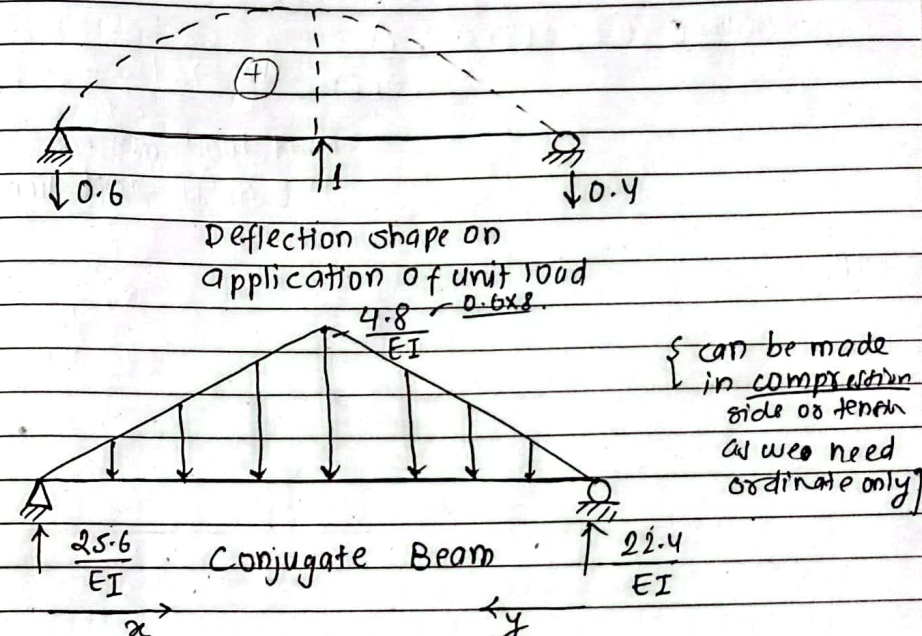
$$6m = \frac{72}{EI}$$

$$8m = \frac{640}{9EI}$$

ILD ordinate = $\frac{\Delta}{M_B}$



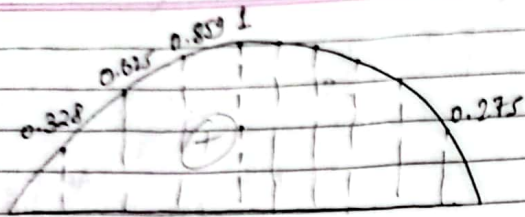
Release support at B & apply unit load



$$M_x = \frac{25.6}{EI}x - \frac{1}{2} \times \left(\frac{4.8}{EI}\right) \times \left(\frac{x}{8}\right) \times \frac{x}{3}$$

$$M_y = \frac{22.4}{EI}y - \frac{1}{2} \times \left(\frac{4.8}{EI}\right) \times \left(\frac{y}{12}\right) \times y \times \frac{y}{3}$$

For x, x=0m → 0	For y, y=0 = 0
x=2m → 50.4/EI	y=2 = 42.27/EI
x=4m → 96/EI	y=4 = 85.33/EI
x=6m → 132/EI	y=6 = 120/EI
x=8m → 153.6/EI	y=8 = 145.07/EI
	y=10 = 157.33/EI
	y=12 = 153.6/EI



ILD for R_B

Ordinate of ILD = Δ

$\delta_{BB} \rightarrow \Delta_B \rightarrow \frac{153.6}{EI}$

unit load लागेको

चाडको deflection ले divide

R_C

M_B

release

Put internal hinge but

do not remove

roller support

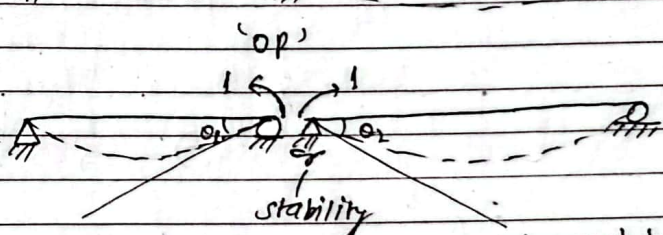
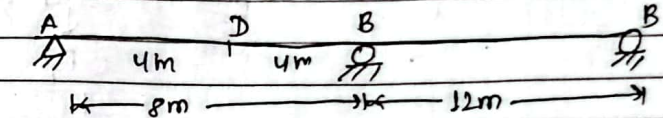
roller support

इतिहास reaction

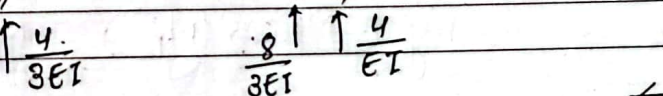
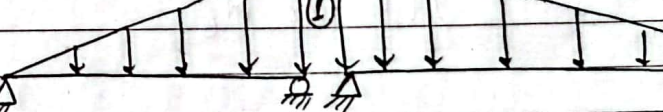
नि release होई

but छाड्ने गर्नु

पर्छ



Moment in real beam 1 & so, SF in conjugate beam



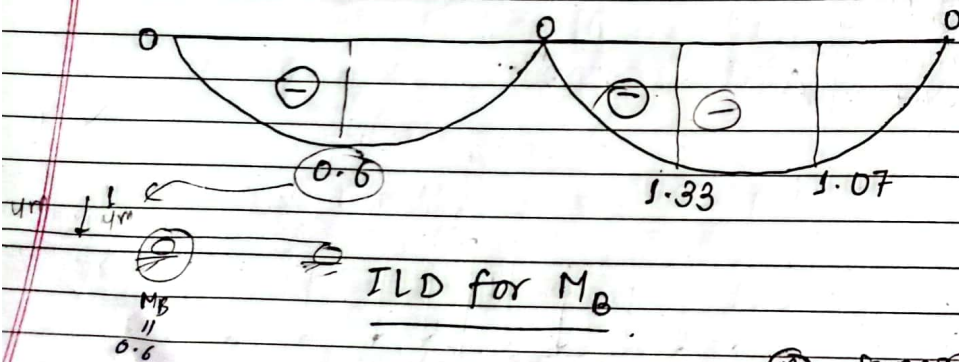
Conjugate beam

$$M_a = \frac{4}{3EI} x - \frac{1}{2} \times \frac{x}{8EI} \times x \times \frac{x}{3}$$

$$M_y = \frac{2}{EI} y - \frac{1}{2} \times \left(\frac{y}{12} \times y \times \frac{y}{3} \right) \times \frac{1}{EI}$$

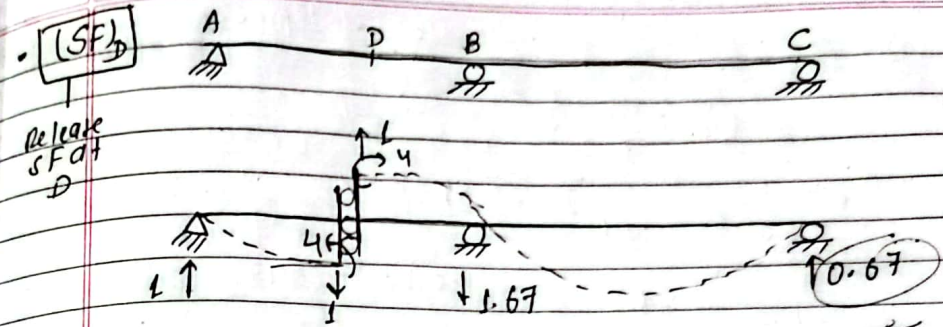
① A $x=0 \rightarrow 0$ At $y=0 \rightarrow 0$
 $x=4 \rightarrow 4/EI$ $y=4 \rightarrow 7.11/EI$
 $x=8 \rightarrow 0$ $y=8 \rightarrow 8.89/EI$
 $y=12 \rightarrow 0/EI$

Divide by $\rightarrow \delta_B \rightarrow$ Slope
 $\downarrow$
 SF_B
 $\downarrow$
 $\frac{8}{3EI} + \frac{4}{EI} = \frac{20}{3EI}$

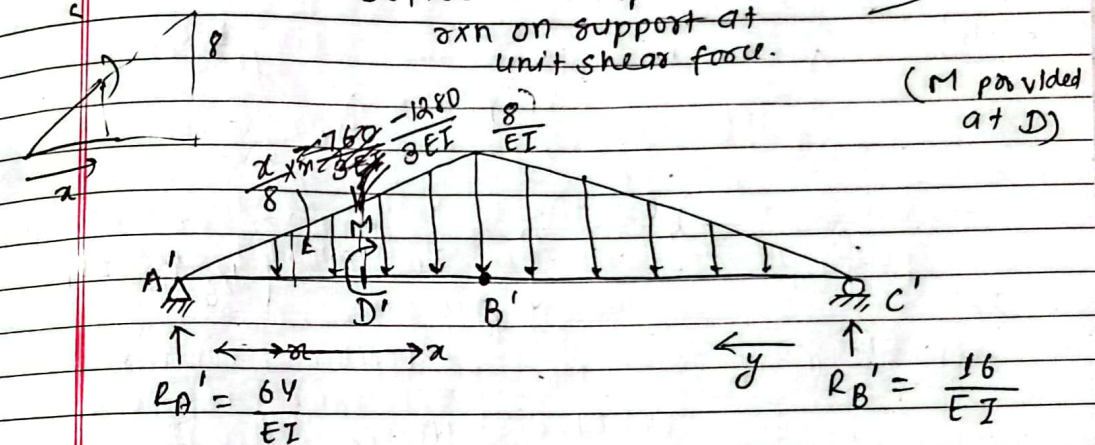


ILD for M_B
 why $\ominus$? $\rightarrow M_B$
 support moment -ve

Support moment always -ve



Deflection shape & rxn on support at unit shear force.



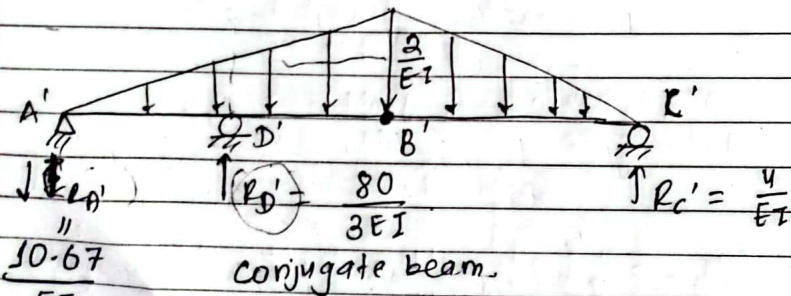
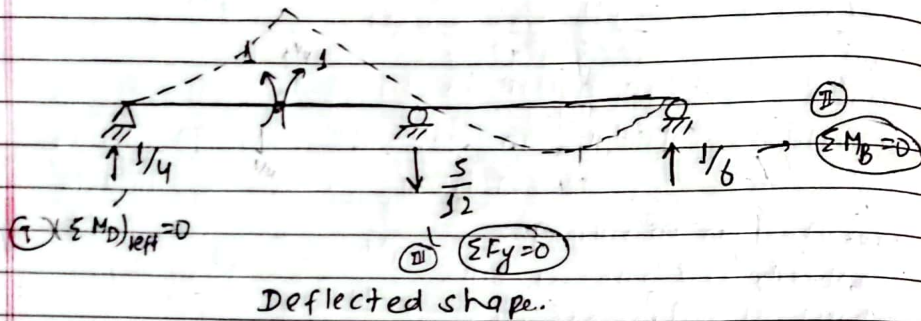
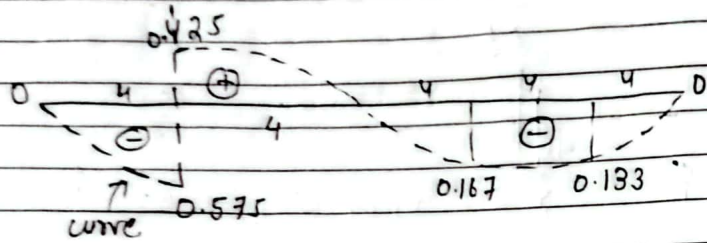
$$\left\{ \begin{array}{l} \sum M_{B'} = 0 \\ \sum M_B = 0 \end{array} \right.$$

$$M_a = \frac{64}{EI} x - \frac{1}{2} \times \left(\frac{8}{8EI} \times x \right) \times \frac{x}{3} \times \frac{1}{3EI}$$

$$M_y = \frac{16}{EI} y - \frac{1}{2} \times \left(\frac{8}{12} \times y \times \frac{y}{3} \right) \times \frac{1}{EI}$$

At $x=0 = 0$ At $y=0 \rightarrow 0$
 $(x=4)_{left} = 254.67/EI$ $y=4 \rightarrow 56.89/EI$
 $(x=8)_{left} = -172/EI$ $y=8 \rightarrow 71.11/EI$
 $(x=8)_{right} = 0$ $y=12 \rightarrow 0$

Divide by BM at D. of Conj beam for ILO.



$$\left\{ \begin{array}{l} (\Sigma M_{B'})_{left} = 0 \\ \& \Sigma F_y = 0 \end{array} \right\}$$

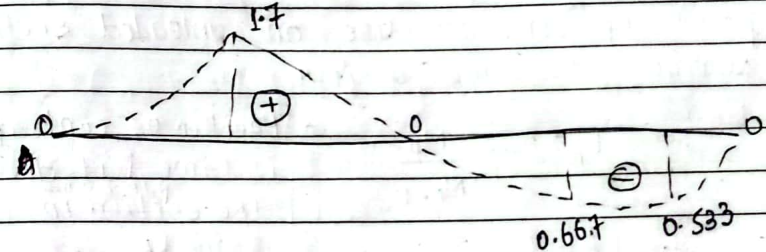
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$$M_a = -\frac{10.67}{EI} x + \frac{80}{3EI} (x-4)^2 - \frac{1}{2} \times \left(\frac{2}{EI \times 8} \right) \frac{x^3}{3}$$

$$M_y = \frac{4}{EI} y - \frac{1}{2} \times \left(\frac{2}{12EI} \right) \times \frac{y^3}{3}$$

At $x=0 \rightarrow 0$ At $y=0 \rightarrow 0$
 $(x=4) \rightarrow -45.35/EI$ $y=4 \rightarrow 14.22/EI$
 $x=8 \rightarrow 0$ $y=8 \rightarrow 17.78/EI$
 $y=12 \rightarrow 0$

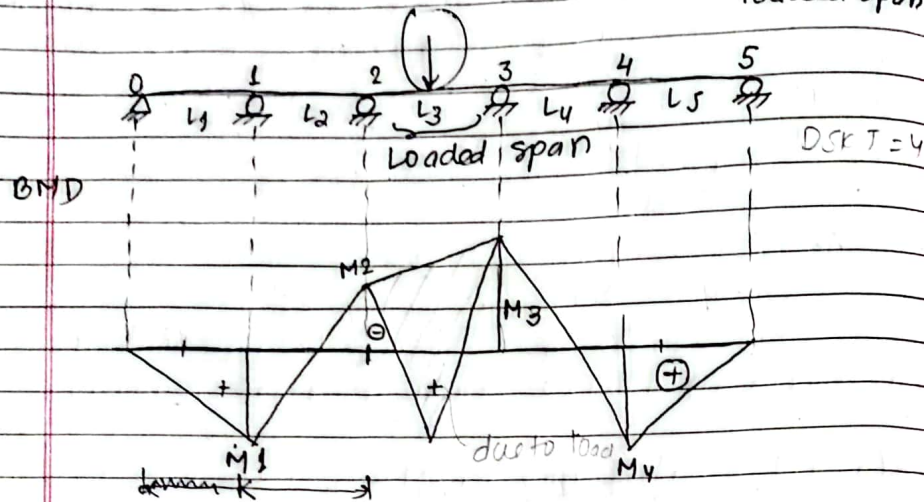
Divide by (R_D')



ILD for M_D

TOS-I ILD * Focal point Ratio

continuous beam → ^{Support area} loaded span → span having load
unloaded span → other than loaded span.



even if mag. & point of action of load changes, point of contraflexure on unloaded section remains at same position.

$$K_n = - \frac{M_n}{M_{n-1}}$$

* Position of contraflexure is same & it divides the section in fixed ratio K_n

↑ loaded span को left निको

$$K_n' = - \frac{M_{n-1}}{M_n}$$

— loaded span को right निको.

eg:- $K_2 = - \frac{M_2}{M_1}$ & $K_4' = - \frac{M_3}{M_4}$

→ In continuous beam there are points of contraflexure on unloaded spans whose positions are unchanged even if the position of load on loaded span changes these points are called focal points and they divide the span into fixed ratio which are equal to the ratio of moments on either side of the span and are called focal point ratios. Depending upon the position of loaded span, the focal point ratios are left moment focal point ratio and right moment focal point ratio when the loaded span is on right side or on left side respectively.

Left focal point ratios are denoted by K_n

$$K_n = - \frac{M_n}{M_{n-1}}$$

Right focal point ratios are denoted by K_n'

$$K_n' = - \frac{M_{n-1}}{M_n}$$

$$K_1 = - \frac{M_1}{M_0} = - \frac{M_1}{0} = \infty$$

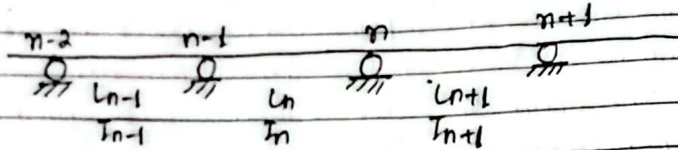
$$K_5' = - \frac{M_4}{M_5} = - \frac{M_4}{0} = \infty$$

{ Rt. focal pt ratio $\frac{L_{n-1}}{L_n}$ $\frac{L_n}{L_{n+1}}$ }
 { Lt. focal pt ratio $\frac{L_{n-1}}{L_n}$ }

Left focal point ratio

(left to support $n-1$ eq. $\frac{L_{n-1}}{L_n}$
 loaded span $n-1$ to n assume $n-1$

Unloaded



- Assume loaded span on right side,
 writing three moment equation for support $(n-1)$

$\frac{M_{n-2}}{I_{n-1}} + 2M_{n-1} \left(\frac{L_{n-1}}{I_{n-1}} + \frac{L_n}{I_n} \right) + M_n \left(\frac{L_n}{I_n} \right) = 0$

[Since no loading, no BMD]

divide

or, $\frac{M_{n-2}}{M_{n-1}} \cdot \frac{L_{n-1}}{L_n} + 2 \left(\frac{L_{n-1}}{L_n} + \frac{L_n}{L_{n+1}} \right) + \frac{M_n}{M_{n-1}} \left(\frac{L_n}{L_{n+1}} \right) = 0$

where, $L_{n-1}' = \frac{L_{n-1}}{I_{n-1}}$

We know, left focal point ratio is

$\frac{1}{k_n} = - \frac{M_n}{M_{n-1}}$
 $\frac{1}{k_{n-1}} = - \frac{M_{n-1}}{M_{n-2}}$

Thus,

or, $-\frac{1}{k_{n-1}} L_{n-1}' + 2(L_{n-1}' + L_n') - k_n L_n' = 0$

Divide by L_n'

or, $-\frac{1}{k_{n-1}} \frac{L_{n-1}'}{L_n'} + 2 \frac{L_{n-1}'}{L_n'} + 2 - k_n = 0$

or, $k_n = -\frac{1}{k_{n-1}} \times \frac{L_{n-1}'}{L_n'} + 2 \left[\frac{L_{n-1}'}{L_n'} + 1 \right]$

or, $k_n = 2 + \frac{L_{n-1}'}{L_n'} \left(2 - \frac{1}{k_{n-1}} \right)$ (1)

For fixed support, $k_n = 2$

For right focal point ratio
 Assume loaded span on left,

(2)

writing three moment equation for support n

$M_{n-1} \cdot \frac{L_n}{I_n} + 2M_n \left[\frac{L_n}{I_n} + \frac{L_{n+1}}{I_{n+1}} \right] + M_{n+1} \cdot \frac{L_{n+1}}{I_{n+1}} = 0$

or, $\frac{M_{n-1}}{M_n} \cdot \frac{L_n}{L_{n+1}} + 2 \left(\frac{L_n}{L_{n+1}} + \frac{L_{n+1}}{L_{n+1}} \right) + \frac{M_{n+1}}{M_n} \cdot \frac{L_{n+1}}{L_{n+1}} = 0$

We know, for right focal point ratio is

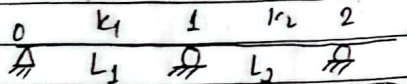
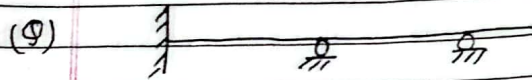
$k_n' = - \frac{M_{n-1}}{M_n}$; $k_{n+1}' = - \frac{M_{n+1}}{M_{n+1}}$

or, $-k_n' \cdot L_n' + 2(L_n' + L_{n+1}') - \frac{1}{k_{n+1}'} \cdot L_{n+1}' = 0$

Dividing by L_n'

or, $-k_n' + \left(2 + \frac{L_{n+1}'}{L_n'} \right) - \frac{1}{k_{n+1}'} \cdot \frac{L_{n+1}'}{L_n'} = 0$

or,
$$K_n' = 2 + \frac{L_{n+1}'}{L_n'} \left[2 - \frac{1}{K_{n+1}'} \right]$$



$I_1 = \infty$ $\left(\frac{L_1}{I_1} = 0 \right)$ $\left(\frac{L_2}{I_2} = \infty \right)$

$\frac{L_1}{I_1} = \infty$

$K_1 = \frac{M_1}{M_0} = \infty$

$$K_n = 2 + \frac{L_{n-1}'}{L_n'} \left[2 - \frac{1}{K_{n-1}'} \right]$$

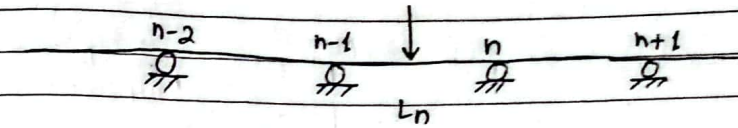
So, $K_2 = 2 + \frac{L_1'}{L_2'} \left[2 - \frac{1}{\infty} \right]$

$= 2 + \frac{0}{L_2'} \left(2 - \frac{1}{\infty} \right)$

$= 2$

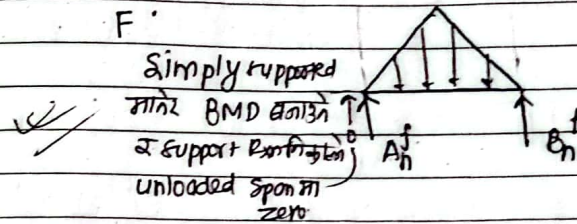
$-\frac{M_2}{M_1} = K_2 \Rightarrow [M_2 = -K_2 \cdot M_1]$

Derivation of formula for support moments on loaded span



Loaded span $\rightarrow n$

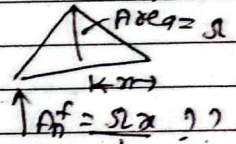
Support moments $\rightarrow n-1$ & n are unknown (left) (right) $\{ M_{n-1} \text{ & } M_n \}$



Writing Three moment eqⁿ for $n-1$,

or
$$\frac{M_{n-2}}{L_{n-1}'} + 2M_{n-1} \left[\frac{L_{n-1}'}{L_{n-1}'} + \frac{L_n'}{L_{n-1}'} \right] + M_n \frac{L_n'}{L_{n-1}'} = -\frac{6 A_n^f}{I_n}$$

$\left(\frac{M_{n-2}}{L_{n-1}'} = -\frac{M_{n-1}}{L_{n-1}'} \right) \{ M_{n-1} \text{ & } M_n \text{ चारिने } M_{n-2} \text{ न चारिने} \}$



or
$$\frac{-M_{n-1} \times L_{n-1}'}{L_{n-1}'} + 2M_{n-1} \cdot L_{n-1}' + 2M_{n-1} L_n' + M_n L_n' = -\frac{6 A_n^f}{I_n}$$

or
$$M_{n-1} \left[2 + \frac{L_{n-1}'}{L_n'} \left[2 - \frac{1}{K_{n-1}'} \right] \right] + M_n = -\frac{6 A_n^f}{L_n' \cdot I_n}$$

or
$$\left[M_{n-1} \cdot K_n + M_n = -\frac{6 A_n^f}{L_n'} \right] \text{--- (1)}$$

Support n

$$M_{n-1} \cdot L'_{n-1} + 2M_n [L'_n + L'_{n+1}] + 3M_{n+1} \cdot L'_n = -\frac{6B_n^f}{I_n}$$

$$\text{or, } M_{n-1} L'_n + 2M_n L'_n + 2M_n L'_{n+1} - \frac{M_n}{K'_{n+1}} L'_n = -\frac{6B_n^f}{I_n}$$

$$\text{or, } M_{n-1} + M_n \left[2 + \frac{L'_{n+1}}{L'_n} \left(2 - \frac{1}{K'_{n+1}} \right) \right] = -\frac{6B_n^f}{L_n}$$

$$\text{or, } \begin{cases} M_{n-1} + M_n K'_n = -\frac{6B_n^f}{L_n} \quad \text{--- (2)} \end{cases}$$

$$\begin{cases} M_{n-1} \cdot K_n + M_n = -\frac{6A_n^f}{L_n} \quad \text{--- (1)} \end{cases}$$

$$M_{n-1} + \left(-\frac{6A_n^f}{L_n} - M_n \cdot K_n \right) \cdot K'_n = -\frac{6B_n^f}{L_n}$$

$$M_{n-1} = \frac{-\frac{6B_n^f}{L_n} + \frac{6A_n^f}{L_n} K'_n}{(1 - K_n K'_n)}$$

$$\left\{ M_{n-1} = -\frac{6}{L_n} \left[\frac{B_n^f - K'_n A_n^f}{1 - K_n K'_n} \right] \right\} \text{--- (3)}$$

$$M_{n-1} = -\frac{6}{L_n} \left[\frac{K'_n A_n^f - B_n^f}{K_n K'_n - 1} \right]$$

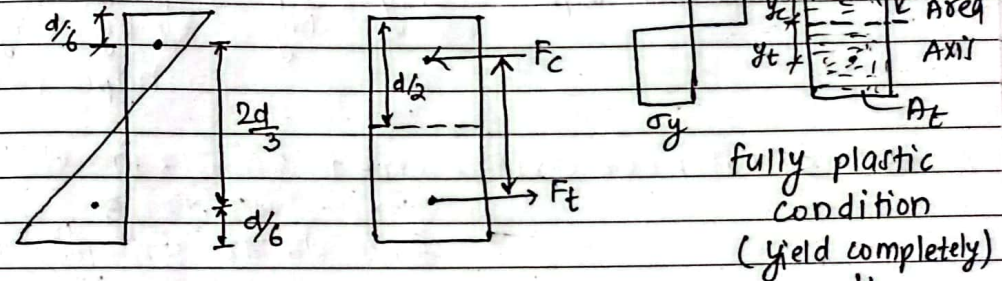
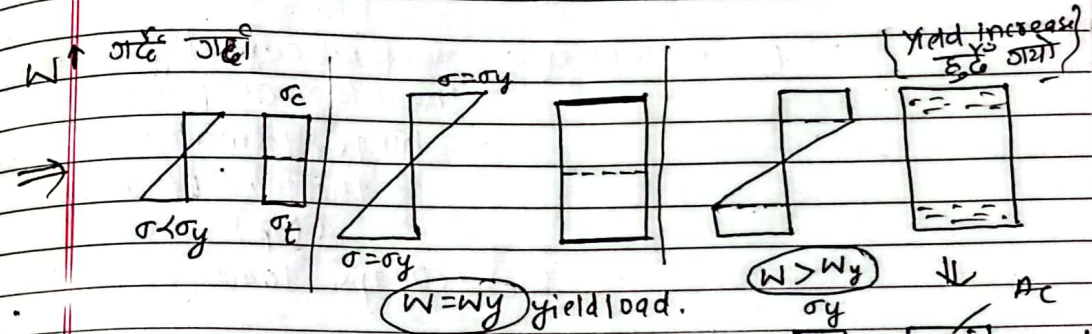
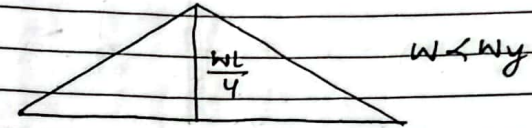
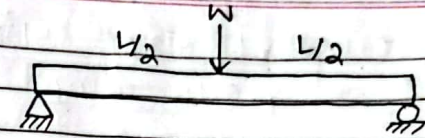
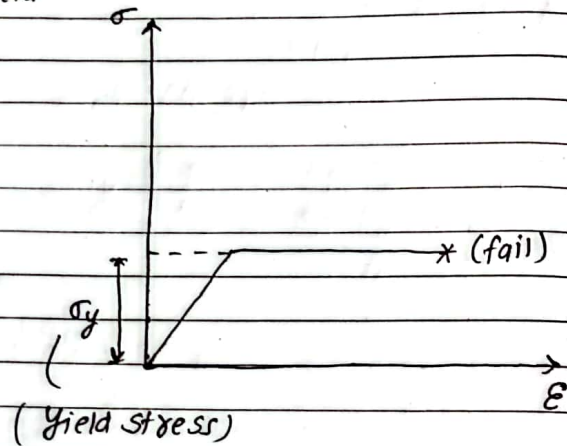
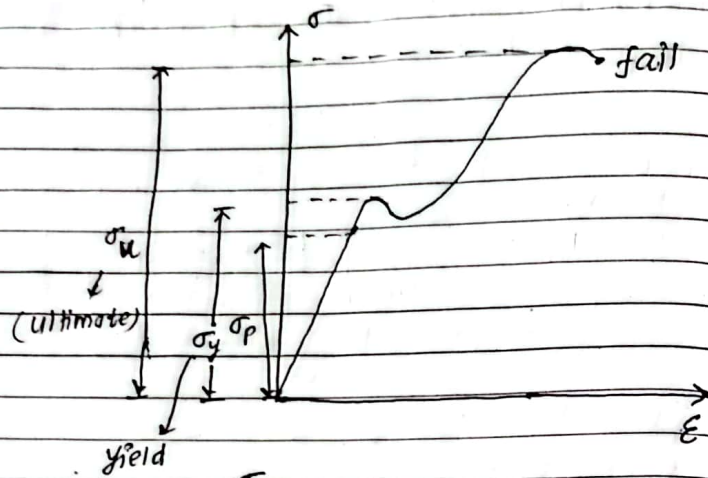
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$$M_n = -\frac{6A_n^f}{L_n} - \left[-\frac{6}{L_n} \left(\frac{B_n^f - K'_n A_n^f}{1 - K_n K'_n} \right) \right] K_n$$

$$= -\frac{6}{L_n} [A_n^f - \frac{B_n^f - K'_n A_n^f}{1 - K_n K'_n} K_n]$$

$$\left\{ M_n = -\frac{6}{L_n} \left[\frac{B_n^f K_n - A_n^f}{K_n K'_n - 1} \right] \right\} \text{--- (4)}$$

5. Plastic Analysis/Plastic Theory



fully plastic condition
(Yield completely)

load is collapse
load & moment
is plastic moment
capacity

$$\sigma = \frac{M y}{I}$$

$$= \left(\frac{\sigma_y + 0}{2} \right) \times \left(\frac{b d}{2} \right) \times \frac{2d}{3} \times \frac{b d^3}{12}$$

→ collapse होने के लगे load बढ़ते हैं hinge बनती है
behaves like → plastic hinge.



Plastic moment → moment capacity of the stage of plastic hinge formation and at the load of collapse (M_p)

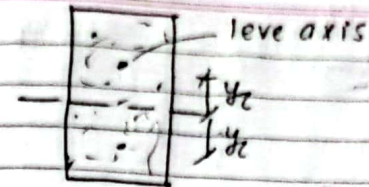
w_c → collapse load.

→ Ratio of M_p and $M_y = \frac{M_p}{M_y} = \frac{Z_p \cdot \sigma_y}{Z_y \cdot \sigma_y} = \frac{Z_p}{Z_y}$
= shape factor

Equal Area axis → area दूनों तिरको बराबर

→ stress बराबर है, force बराबर हूँ फिर भी बराबर हूँ पथों

यानी about मा force बराबर हूँ



$$A_c = A_t$$

$$\text{or, } F_c = A_c \cdot \sigma_y$$

$$F_t = A_t \cdot \sigma_y$$

$$\text{So, } F_c = F_t$$

$$M_p = (F_c \times y_c + F_t \times y_t)$$

$$= \frac{A}{2} \sigma_y (y_c + y_t)$$

Now,

$$Z_p = \frac{A}{2} (y_c + y_t)$$

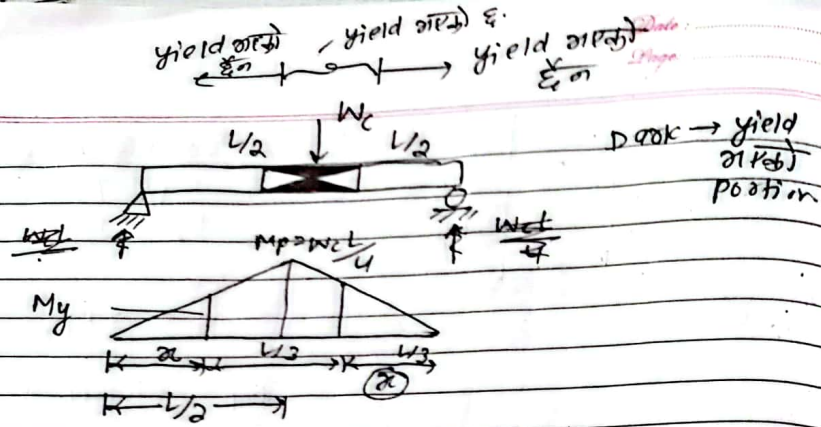
Rectangular

$$Z_p = \frac{bd}{2} \left(\frac{d}{4} + \frac{d}{4} \right) = \frac{bd^2}{4}$$

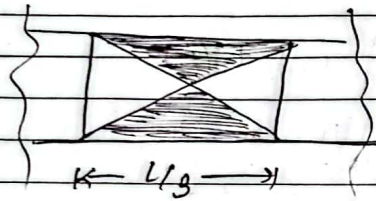
$$\text{For rectangular, } Z_y = \frac{bd^2}{6}$$

Thus,

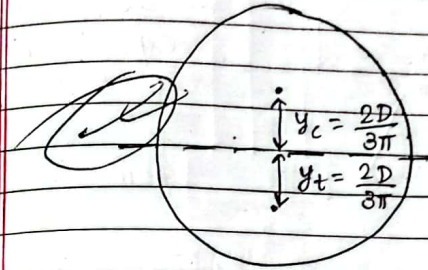
$$\text{Shape factor} = \frac{Z_p}{Z_y} = \frac{6}{4} = \frac{3}{2} = \underline{\underline{1.5}}$$



$a = L/3$ form ~~part~~ shape factor.



* Shape factor for circular section
In circle, neutral axis and equal area axis is same.



$$Z_y = \frac{I}{y_{max}}$$

$$= \frac{\pi D^4}{64} \cdot \frac{D/2}{\frac{32}{32}}$$

$$= \frac{\pi D^3}{32}$$

$$Z_p = \frac{A}{2} (y_c + y_t)$$

$$= \frac{\pi D^2}{8} \left(\frac{2D}{3\pi} + \frac{2D}{3\pi} \right)$$

$$= \frac{\pi D^3}{8}$$

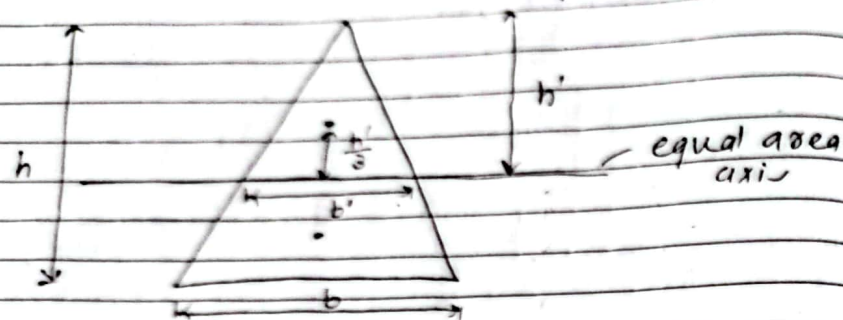
$$\text{Shape factor} = \frac{Z_p}{Z_y}$$

$$= \frac{\frac{\pi D^3}{8}}{\frac{\pi D^3}{32}}$$

$$= \frac{16}{8\pi}$$

$$= \underline{\underline{1.69}}$$

Shape factor for triangular section



$$\frac{b}{b'} = \frac{h}{h'}, \quad b' = \frac{bh'}{h}$$

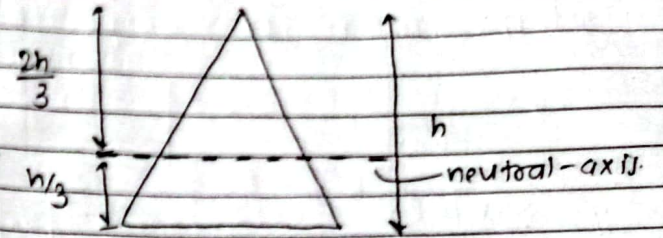
$$\frac{1}{2} b' h' = \frac{A}{2} = \frac{bh}{4};$$

$$h' = \frac{h}{\sqrt{2}}, \quad b' = \frac{b}{\sqrt{2}}$$

$$Z_p = \frac{A}{2} (y_c + y_t)$$

$$= \frac{bh}{2 \times 2} \left(\frac{h'}{3} + \frac{h-h'}{3} \left(\frac{b'+2b}{b'+b} \right) \right)$$

$$= \frac{bh}{4} \left[\frac{h}{3\sqrt{2}} + \frac{h-h/\sqrt{2}}{3} \left(\frac{b/\sqrt{2}+2b}{b/\sqrt{2}+b} \right) \right]$$



$$Z_y = \frac{bh^2}{24}$$

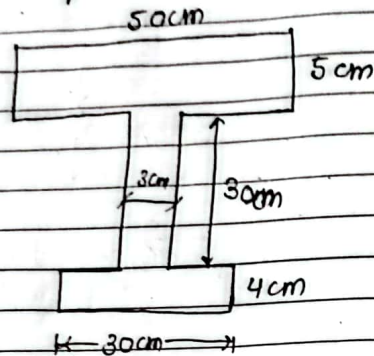
$$\text{Shape factor} = \frac{Z_p}{Z_y}$$

Shape factor for composite section

$$Z_y = \frac{I}{Y_{max}}$$

$$\bar{y} = \frac{30 \times 4 \times 2 + 30 \times 3 \times 19 + 50 \times 5 \times 36.5}{30 \times 4 + 30 \times 3 + 50 \times 5}$$

$$= 24.076 \text{ cm (base)}$$



$$\text{top: } 30 + 4 + 5 - 24.076 = 14.924 \text{ cm}$$

→ More from base

$$I = \frac{30 \times 4^3}{12} + 30 \times 4 (24.076 - 2)^2$$

$$+ \frac{3 \times 30^3}{12} + 3 \times 30 (24.076 - 19)^2$$

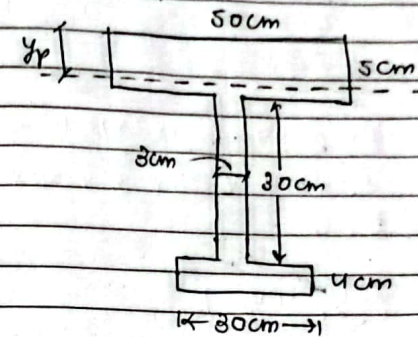
$$+ \frac{50 \times 5^3}{12} + 50 \times 5 (36.5 - 24.076)^2$$

$$= 106820 \text{ cm}^4$$

Now,

$$Z_y = \frac{I}{Y_{max}} = \frac{106820}{24.076} = 4436.81 \text{ cm}^3$$

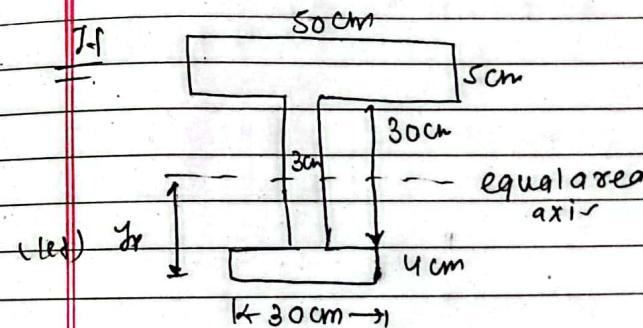
Equal area ⇒



मात्रिको portion होई द. 20, assume equal area axis in upper part.

$$y_p \times 50 = (5 - y_p) \times 50 + 30 \times 3 + 30 \times 4$$

$$y_p = 4.6 \text{ cm}$$



$$(y_p - 4) \times 3 + 30 \times 4 = (30 - y_p - 4) \times 3 + 50 \times 5$$

$$\Rightarrow y_p = 40.667$$

(not possible)

(Centroid of Now,
all parts)
position

$$y_c = \frac{4.6}{2} = 2.3 \text{ cm}$$

(Centroid of
steel
position)

$$y_s = \frac{0.4 \times 50 \times 0.2 + (30 \times 3 \times 15.4) + 30 \times 4 \times 32.4}{50 \times 0.4 + 30 \times 3 + 30 \times 4}$$

$$= 22.95 \text{ cm}$$

$$Z_p = \frac{A}{2} (y_c + y_s)$$

$$= \frac{(50 \times 5 + 30 \times 3 + 30 \times 4)}{2} [2.3 + 22.95]$$

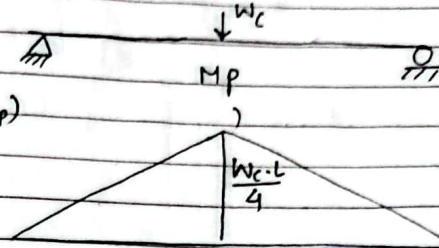
$$= 5807.5 \text{ cm}^3$$

$$\text{Shape factor} = \frac{Z_p}{Z_y} = \frac{5807.5}{4436.1} = 1.3$$

Determination of collapse load or plastic moment capacity.

Static Method:

BMD draw
at max BM (Mp)
find Wc



$$M_p = \frac{W_c L}{4}$$

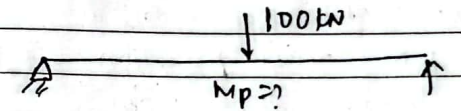
$$\Rightarrow W_c = \frac{4 M_p}{L}$$

Load factor = 1.5

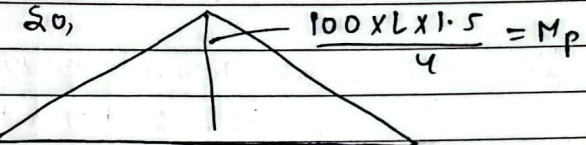
then multiply load with load factor to find Mp

eg:

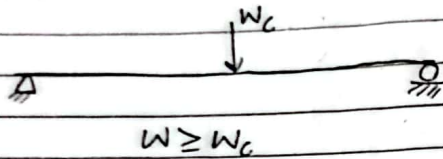
$$\text{Max BM} = \frac{W_c L \times \text{load factor}}{4}$$



$$M_p = \frac{(1.5 \times 100) L}{4}$$



(2) Kinematic Method



Collapse structure



required

(Min.) Number of Hinge^{required} for collapse = $DSI + 1$ & [Hinge production].

- Position of plastic hinges

(1) → pt. load act जवको लाउंछा

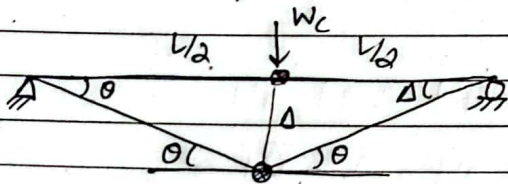
(2) $\rightarrow$ Max.^m BM at (if no pt. load)

(3) Fixed support भएको छिटो

(4) Rigid joint of frame π

(5) Junction of different cross section (I-vary)

∴ Above eq:- $DRT = 0$ So, Min. no. of hinge = 1
(at position of pt. load)



(We know,

$$W_{\text{external}} = W_{\text{internal}}$$

(due to applied load) (rotation)

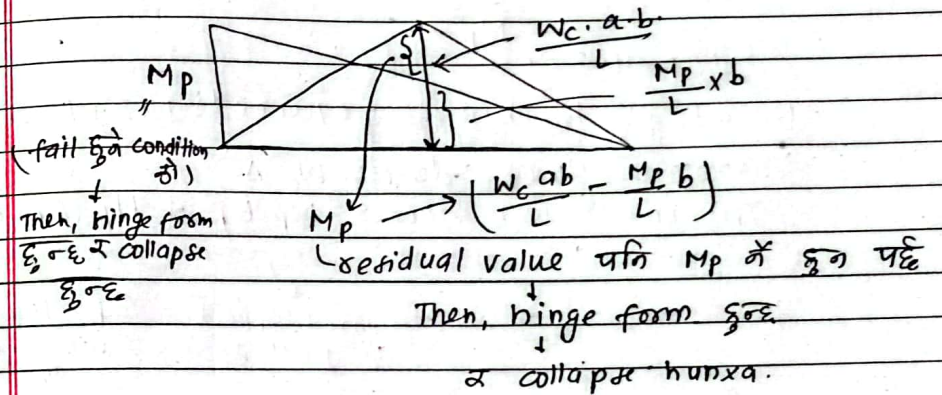
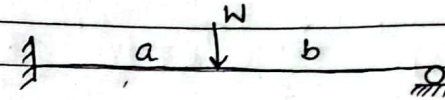
$$\text{or, } W_C \cdot \Delta = M_P (\theta + \theta)$$

$$\sin \theta = \frac{\Delta}{L/2} = \frac{2\Delta}{L}$$

~~50,~~

$$W_c \cdot \Delta = M_p \times 2 \times 2 \frac{\Delta}{L}$$

$$\Rightarrow [W_c = \frac{4Mp}{L}] \text{ (u)}$$



Min. no. of hinge required for collapse = $1+1 = 2$

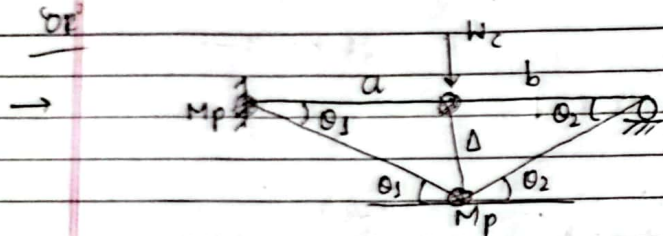
(20, दूसरे part में hinge formation
गणपति मात्र collapse हुआ)

fixed support & load acting
pt. C (where value of
moment is M_p)

$$M_p = \frac{w_c ab}{L} - \frac{M_p}{L} \times b$$

$$w_c = M_p \left(1 + \frac{b}{L} \right) \times \frac{L}{ab}$$

$$\int w_c = \frac{M_p (L+b)}{ab} \quad \text{--- (u)}$$



$$DST = 4 - 3 = 1$$

$$\text{No. of plastic hinges required} = DST + 1 = 2$$

$$\text{External work done} = W_c \cdot \Delta$$

$$\text{Internal work done} = M_p \theta_1 + M_p (\theta_1 + \theta_2)$$

fixed support
st position

pt. load
moving point
on

Then,

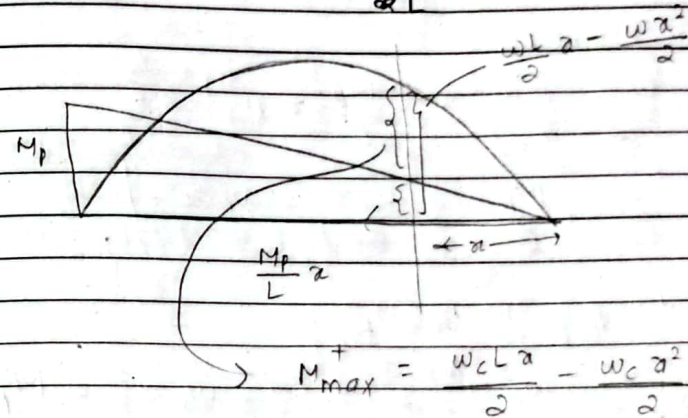
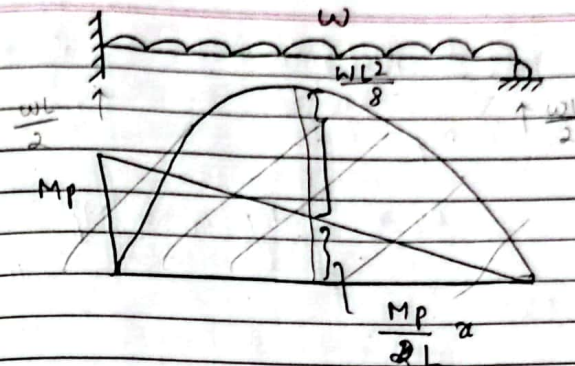
$$W_c \Delta = M_p \cdot \theta_1 + M_p (\theta_1 + \theta_2)$$

$$\text{or } W_c \cdot \Delta = M_p \cdot \frac{\Delta}{a} + M_p \left(\frac{\Delta}{a} + \frac{\Delta}{b} \right)$$

$$\int w_c = \frac{M_p}{a} + M_p \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$\text{--- (u)} \quad \int w_c = M_p \left[\frac{2}{a} + \frac{1}{b} \right]$$

(9)



$$M_{max} = \frac{w_c L a}{2} - \frac{w_c a^2}{2} - \frac{M_p a}{L}$$

$$\frac{dM_{max}}{da} = 0 \quad (\text{for max}^e \text{ +ve B.M.})$$

$$\text{or } \frac{w_c L}{2} - w_c a - \frac{M_p}{L} = 0 \quad \text{--- (1)}$$

$$\text{or } a = \frac{\frac{w_c L}{2} - \frac{M_p}{L}}{w_c} = \frac{w_c L^2 - 2M_p}{2Lw_c}$$

For collapse condition

$$M_{max}^+ = M_p$$

$$\text{or } \frac{w_c L^2}{2} - \frac{w_c a^2}{2} - \frac{M_p a}{L} = M_p \quad \text{--- (i)}$$

$$\text{or } \frac{w_c L}{2} - \frac{w_c a}{L} = \frac{M_p}{L}$$

From (i),

$$\frac{w_c L}{2} - w_c a - \frac{M_p}{L} = 0$$

$$\left[\frac{M_p}{L} = \frac{w_c L}{2} - w_c a \right] \quad \text{--- (ii)}$$

keeping it in (i)

$$\frac{w_c L^2}{2} - \frac{w_c a^2}{2} - \frac{w_c L^2}{2} + w_c L a = M_p$$

$$\text{or } \frac{w_c a^2}{2} = M_p$$

$$\text{or } \left[w_c = \frac{2 M_p}{a^2} \right] \quad \text{--- (iii)}$$

$$w_c = \frac{2 M_p}{a^2}$$

from (iii)

$$\text{or } \frac{w_c a^2}{2} = \frac{w_c L^2}{2} - w_c L a$$

$$\text{or } a^2 + 2 L a - L^2 = 0$$

Solving

$$* [a = 0.414 L]$$

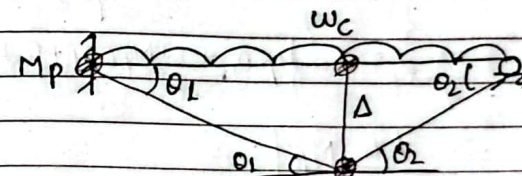
↳ position of hinge from roller support

Now,

$$* \left[w_c = \frac{2 M_p}{a^2} = \frac{2 M_p}{(0.414 L)^2} = \frac{11.67 M_p}{L^2} \right]$$

↳ critical load.

Thus,



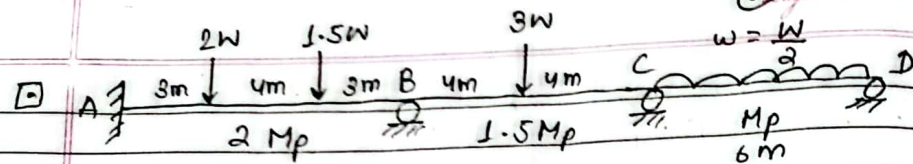
$$* 0.586 L \quad \text{--- (iv)}$$

$$(w_c \cdot L) \left(\frac{\Delta}{2} \right) = M_p \theta_1 + M_p (\theta_1 + \theta_2)$$

or load distributed
E so, Δ पक्षित
pt. load तब sq
 Δ जित

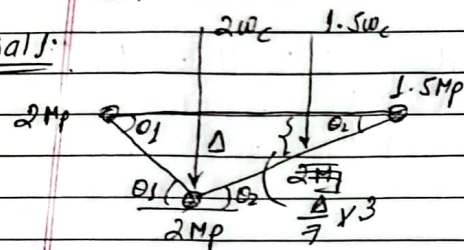
$$\text{or } w_c L \cdot \frac{\Delta}{2} = M_p \cdot \frac{\Delta}{0.586 L} + M_p \left(\frac{\Delta}{0.586 L} + \frac{\Delta}{0.414 L} \right)$$

$$* \left[w_c = \frac{11.67 M_p}{L^2} \right] \quad \text{--- (v)}$$



continuous beam को नि fixed पर्व मान्ने
 (as moment आउने)

Span AB

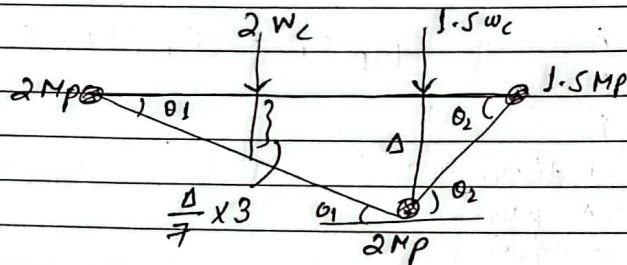


$$\theta_1 = \frac{\Delta}{3}$$

$$\theta_2 = \frac{\Delta}{9}$$

$$2W_c \times \Delta + 1.5W_c \times \frac{\Delta}{3} \times 3 = 2Mp\theta_1 + 2Mp(\theta_1 + \theta_2) + 1.5Mp\theta_2$$

$$\text{or } [W_c = \frac{77}{111} Mp]$$



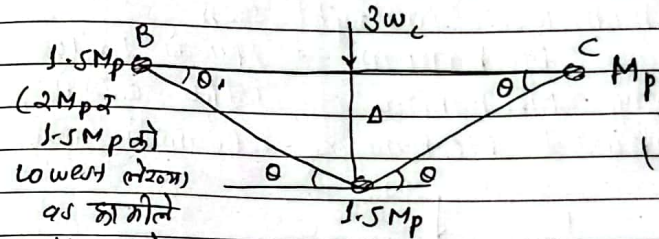
$$2W_c \times \frac{\Delta}{3} \times 3 + 1.5W_c \cdot \frac{\Delta}{9} = 2Mp\theta_1 + 2Mp(\theta_1 + \theta_2) + 1.5Mp\theta_2$$

$$\text{or } [W_c = \frac{73}{99} Mp]$$

Capacity कति → highest रोजने (W_c)
 Load → lowest रोजने (W_c)
 for failure

trial एट भएको $\max W_c$ भएको condition लिने

Span BC



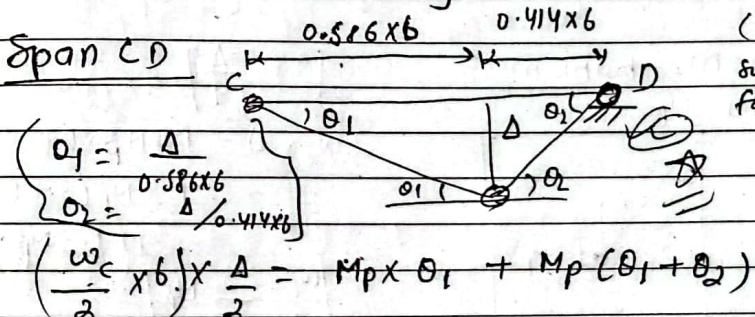
(direction मा small
 value of M_p लिने)

$$(\theta = \frac{\Delta}{9})$$

$$3W_c \times \Delta = 1.5Mp\theta + 1.5Mp(\theta + \theta) + Mp \times \theta$$

$$= [W_c = \frac{11}{24} Mp]$$

Span CD



(D को higher
 support मा हinge
 form भएको छैन)
 ↑
 D मा पढेको

$$\left(\frac{W_c}{2} \times 6 \right) \times \frac{\Delta}{2} = Mp \times \theta_1 + Mp(\theta_1 + \theta_2)$$

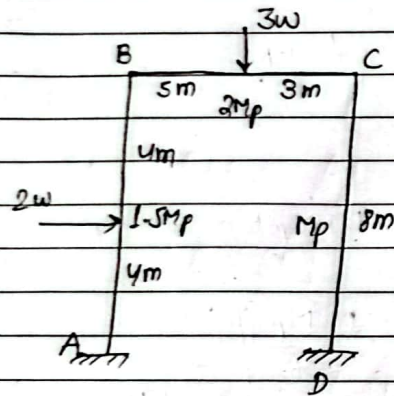
$$\left[\frac{W_c}{2} \times 6 \right] \times \frac{\Delta}{2} = Mp \times \theta_1 + Mp(\theta_1 + \theta_2)$$

Thus, taking lowest value of all,
collapse load, $w_c = \frac{11 M_p}{24} \Rightarrow$

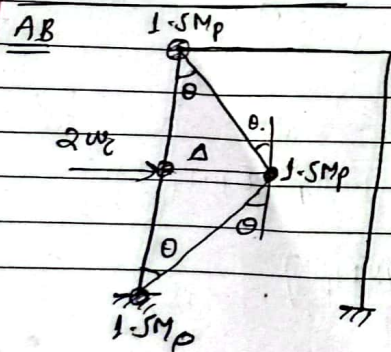
Frame.

3 Mechanisms:- (must be seen)

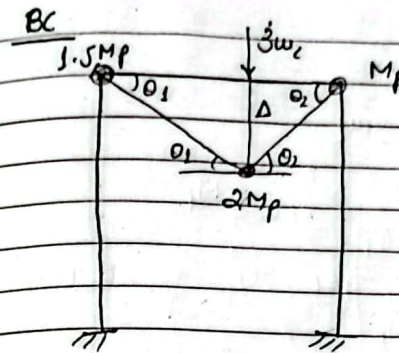
- ① Beam Mechanism $\rightarrow$ span को load लागेको
- ② Sway Mechanism $\rightarrow$ एकतिर हल्लेको
- ③ Combined Mechanism $\rightarrow$ Beam + sway



Beam Mechanism



$$\begin{aligned} \theta &= \frac{\Delta}{4} \\ 2w_c \times \Delta &= 1.5M_p \times \theta + 1.5M_p (\theta + \theta) + 1.5M_p \times \theta \\ \Rightarrow w_c &= \frac{3}{4} M_p \end{aligned}$$

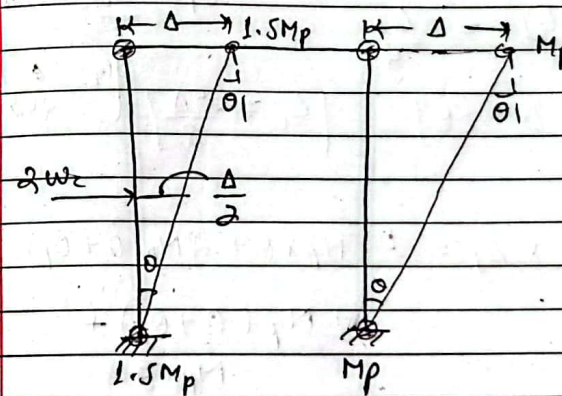


$$\begin{aligned} 3w_c \times \Delta &= 1.5M_p \theta_1 + 2M_p (\theta_1 + \theta_2) + M_p \times \theta_2 \\ \theta_1 &= \frac{\Delta}{5}, \quad \theta_2 = \frac{\Delta}{3} \\ \Rightarrow w_c &= \frac{17}{30} M_p \\ &= 0.566 M_p \end{aligned}$$

Sway Mechanism

(considering inextensible)

$$\text{No. of hinges} = DSI + 1 = 3 + 1 = 4$$

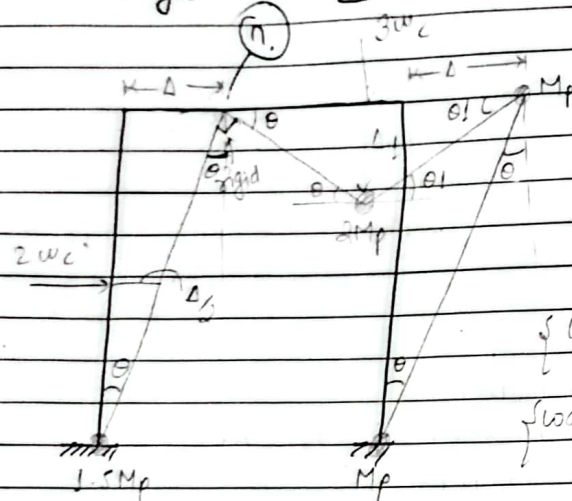


$$\theta = \frac{\Delta}{8}$$

$$\begin{aligned} 2w_c \cdot \frac{\Delta}{2} &= 1.5M_p \times \theta + 1.5M_p \times \theta + M_p \times \theta + M_p \times \theta \\ \Rightarrow w_c &= \frac{5}{8} M_p \\ &= 0.625 M_p \end{aligned}$$

Combined Mechanism

Hinge $\rightarrow$ still 4.



{ load & movement
same dir $\rightarrow$ +ve
{ load & movement
opp dir $\rightarrow$ -ve

$$\theta_1 = \frac{\Delta_1}{3} = \frac{(5\Delta/8)}{3}$$

$$[\theta_1 = \frac{5\Delta}{24}]$$

$$\theta = \frac{\Delta}{8}, \quad \theta_1 = \frac{\Delta_1}{5}$$

$$\theta_1 = \frac{\Delta_1}{3} = \frac{(5\Delta/8)}{3} = \frac{5\Delta}{24}$$

$$\Rightarrow [\Delta_1 = 5\theta]$$

$$[\Delta_1 = \frac{5\Delta}{8}]$$

Thus,

$$2w_c \times \frac{\Delta}{2} + 3w_c \times \Delta_1 = 1.5M_p \times \theta + 2M_p(\theta + \theta_1)$$

$$+ M_p(\theta + \theta_1) +$$

$$M_p \cdot \theta$$

$$2w_c \times \frac{\Delta}{2} + 3w_c \times \frac{5\Delta}{8} = 1.5M_p \times \frac{\Delta}{8} + 2M_p \left(\frac{\Delta}{8} + \frac{5\Delta}{24} \right)$$

$$+ M_p \left(\frac{\Delta}{8} + \frac{5\Delta}{24} \right) + M_p \times \frac{\Delta}{8}$$

$$\Rightarrow w_c = 0.456 M_p$$

Adopting lowest value,

$$\text{collapse load, } w_c = 0.456 M_p$$

#

DST $\Sigma(m'-3)$ ~~Frame~~ Frame : $(DST)_e = 8-3$

SURENDRA SHARMA

 $(DST)_i = 3 \times \text{no. of closed loop} - \text{no. of eqs of conditions}$ Trauss : $(DST)_e = 8-3$ $(DST)_i = m - (2j - 3)$ DKT : Add^l eq^s required to determine independent displacement components.Trauss = $2j - \text{restraints}$ Frame: $3j - \text{restraints}$ (extensible) $3j - \text{restraints} - m$ (inextensible)

Maxwell Reciprocal Theorem.

[PCE & POS]. Linearly elastic str. in static eq^s. $\epsilon [\delta_{ij} = \delta_{ji}]$

Betti's theorem

 $\sum P_i \times \Delta_i' = \sum P_i' \times \Delta_i$ } virtual work. $P_i \& P_i' \rightarrow$ two system of forces $\Delta_i \& \Delta_i' \rightarrow$ displacement in two system of force.

Castigliano's Theorem

(I) $\frac{\partial U}{\partial \Delta_j} = P_j$ } $\rightarrow$ Strain energy, $U = \int \frac{M^2 dx}{2EI}$ (Beam flexural)(II) $\frac{\partial U}{\partial P_j} = \Delta_j$ } $U = \sum \frac{P^2 L}{2AE}$ (Trauss axial) $\rightarrow$ For $DST = 0 \rightarrow$ find θ & Δ $\rightarrow$ For $DST \geq 1 \rightarrow$ find redundant forces or moments

Steps:

- ① Put pt. of deflection (जहाँ deflection हो) (यही pt. में dummy load/moment लगाओ)
- ② Express U in terms of dummy load (या P)

$$U = \int \frac{M^2 dx}{2EI} \text{ 'or' } U = \int \frac{P^2 dx}{2AE}$$

(Dummy load को term में रखें)

$$\Delta = \frac{\partial U}{\partial P}$$
- ③ Put actual value of load P (at given position)
 - ↳ not exist actually $\rightarrow P=0$
 - ↳ Redundant $\rightarrow P=R$
 - ↳ exist $\rightarrow P = \text{value}$
- ④ Find Δ or θ 'or' Redundant

+ve $\rightarrow$ dirⁿ of dummy load

-ve $\rightarrow$ opp. to dirⁿ of dummy load

$\times$ Redundant को posⁿ में dummy load लगाओ

Flexibility

$$[S][X] + [\Delta_L] = [\Delta]$$

Stiffness

$$[K][\Delta] + [P_L] = [P]$$

2. Force Method

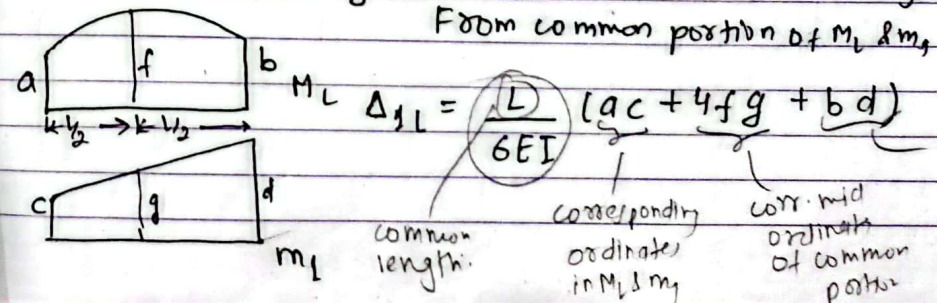
- ① DSI
- ② Primary str.
Find rxns (w/o considering Redundants) (M_L)
- ③ Unit force diagram (for unit value of redundant at each position)
 $m_1, m_2, \dots$
- ④ Find displacement due to load at coordinates assigned,

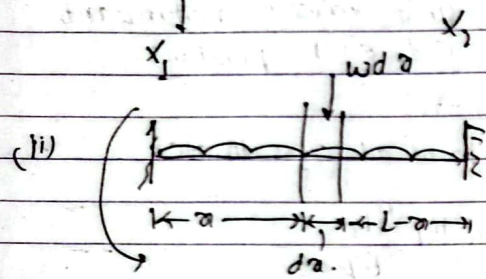
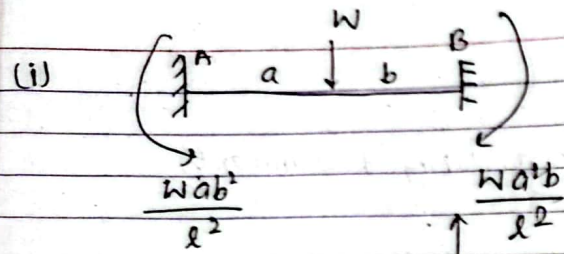
$$\Delta_{1L} = \int M_L m_1 \frac{dx}{EI}$$
- ⑤ Flexibility matrix
 $[S] \rightarrow \delta_{11}, \delta_{22}, \delta_{33}, \delta_{12}, \delta_{21}, \dots$
- ⑥ Solve compatibility eqⁿ

$$[X] = [S^{-1}] \{ [\Delta] - [\Delta_L] \}$$

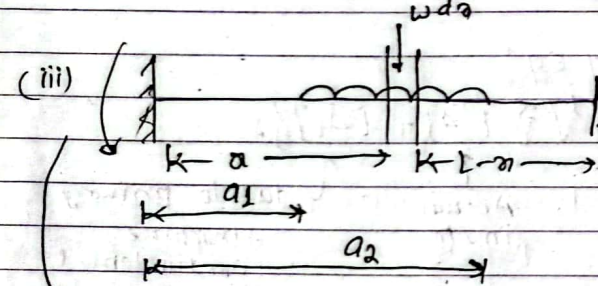
↑ Actual into.
 ↑ due to primary structure (redundants + external loads)
 ↓ removing support

* Note: Graphically:



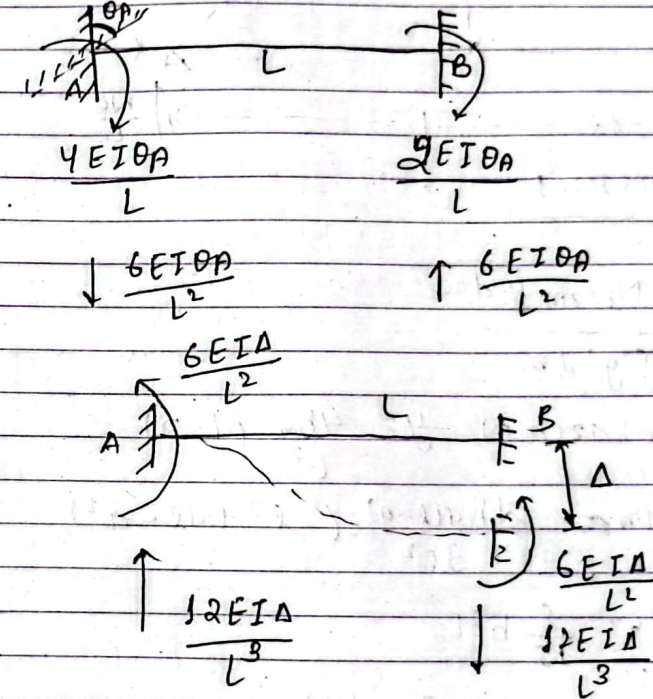


$$\int_0^L \frac{(wda)(L-a)^2}{L^2} = \frac{WL^2}{12}$$



$$M = \int_{a_1}^{a_2} \frac{(wda)(L-a)^2}{L^2}$$

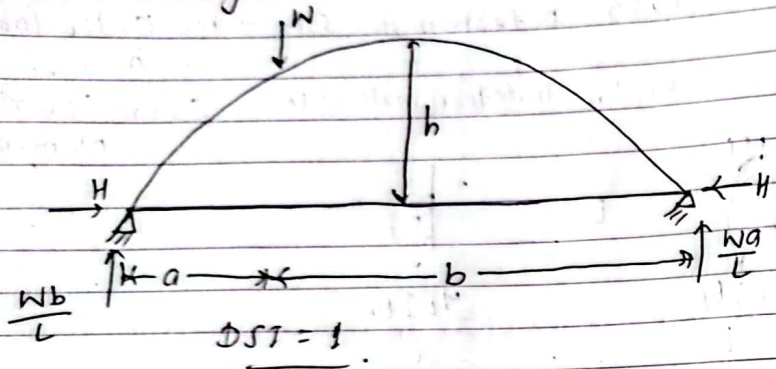
- (iv) Support settlement & rotation
 L In determinate str: $\rightarrow$ no extra load/moment
 L In indeterminate str $\rightarrow$ extra load/moment



TRUSS, $\Delta_{IL} = \frac{\sum P_L P_1 L}{AE}$
 $\delta_{11} = \frac{\sum P_1^2 L}{AE}$

compatibility eqⁿ & solve redundants for

Two hinged arch



$$H = \frac{\int M_{beam} y d\alpha}{\int y^2 d\alpha}$$

Parabolic arch $\Rightarrow y = \frac{4h\alpha}{L^2} (L-\alpha)$

$$\int M_b y d\alpha = \frac{W h a (L-a)}{3L^2} (L^2 + aL - a^2)$$

$$\int y^2 d\alpha = \frac{8}{15} h^2 L$$

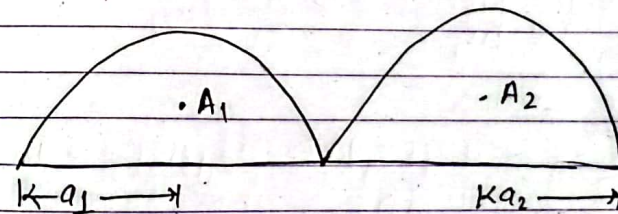
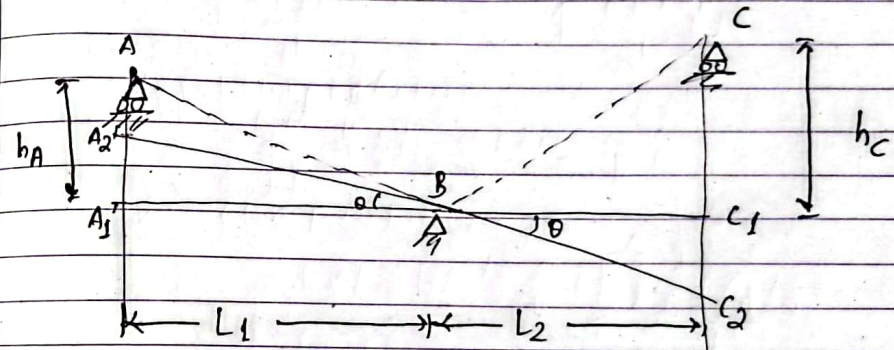
for $a = nL$ ($n \leq 1$)

$$H = \frac{5WnL}{8h} (1 - 2n^2 + n^3)$$

Three moment equation

$$\frac{L_n}{I_n} M_{n-1} + 2 \left[\frac{L_n}{I_n} + \frac{L_{n+1}}{I_{n+1}} \right] M_n + \frac{L_{n+1}}{I_{n+1}} M_{n+1} = -6 \left[\frac{\Omega_n \cdot a_n}{I_n \cdot L_n} + \frac{\Omega_{n+1} \cdot a_{n+1}}{I_{n+1} \cdot L_{n+1}} \right] + \frac{6Eh_{n-1}}{L_{n-1}} + \frac{6Eh_{n+1}}{L_{n+1}}$$

opposite support distance



Free moment dia.
 (M_A, M_B, M_C)
End moments.

$$M_A \left(\frac{L_1}{I_1} \right) + 2M_B \left(\frac{L_1}{I_1} + \frac{L_2}{I_2} \right) + M_C \left(\frac{L_2}{I_2} \right) = - \frac{6A_1 a_1}{I_1 L_1} - \frac{6A_2 a_2}{I_2 L_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_B}{L_2}$$

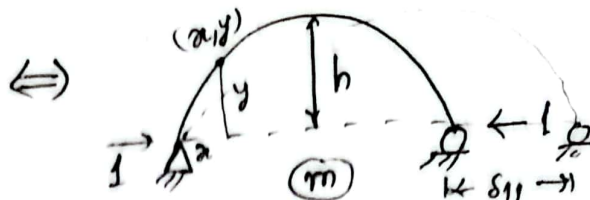
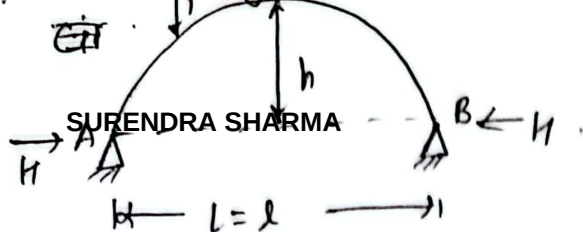
h_A & $h_C \rightarrow$ upward settlement of support A & C wrt. support B.

if fix support then $I = \infty$ assume it.

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s.t. $\frac{L}{I}$ component is zero.

Two hinged Arch



$$H = + \frac{\int \frac{Mm ds}{EI}}{\int \frac{m^2 ds}{EI}}$$

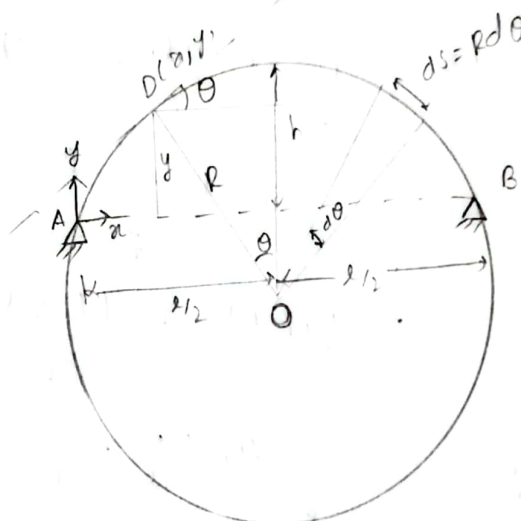
$m \rightarrow l \times y = (-y)$
 $ds \rightarrow$ carried out along axis of arch

$$\left\{ H = - \frac{\int \frac{My ds}{EI}}{\int \frac{y^2 ds}{EI}} \right\}$$

-ve $\rightarrow$ compressive force

• Circular Arch

$$\left[\begin{aligned} R &= \frac{l^2}{8h} + \frac{h}{2} \\ a &= \frac{l}{2} - R \sin \theta \\ R &= R \cos \theta - (R - h) \\ ds &= R d\theta \end{aligned} \right]$$



• Parabolic Arch

$$\begin{aligned} \sec \theta &= ds/dx \\ \Rightarrow ds &= dx \sec \theta \end{aligned}$$

considering 'A' as origin,

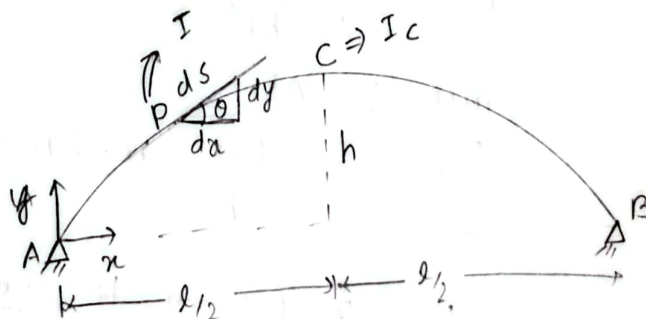
$$y = \frac{4hx}{l^2} (l-x)$$

$$\theta = \frac{dy}{dx} = \frac{4h}{l^2} (l-2x)$$

curve of an arch axis must be parabolic

MOT of ab at any particular section is equal to product of MOT at crown & secant of angle θ

i.e. $I = I_c \sec \theta$ } Thus, for parabolic arch, $ds = dx$



- $z \rightarrow$ Absolute position of W
- $n = \frac{z}{l} \rightarrow$ relative position of W

$z \Rightarrow 0$ to l

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$$M_a = V_A \cdot a = \frac{W a (l - z)}{l}$$

$$(0 \leq a \leq z)$$

$$M_a = V_B \cdot (l - a) = \frac{W z (l - a)}{l}$$

$$(z \leq a \leq l)$$

$$H = \frac{- \int M_y da}{\int y^2 da}$$

$$\begin{aligned} \rightarrow \int_0^l M_y da &= \int_0^z \frac{W a (l - z)}{l} \cdot 4h \left(\frac{a}{l} - \frac{a^2}{l^2} \right) da \\ &+ \int_z^l \frac{W z (l - a)}{l} \cdot 4h \left(\frac{a}{l} - \frac{a^2}{l^2} \right) da \\ &= \frac{W z (l - z) (l^2 + l z - z^2) \cdot h}{3l^2} \end{aligned}$$

$$\rightarrow \int_0^l y^2 da = \int_0^l \left[4h \left(\frac{a}{l} - \frac{a^2}{l^2} \right) \right]^2 da = \frac{8}{15} l h^2$$

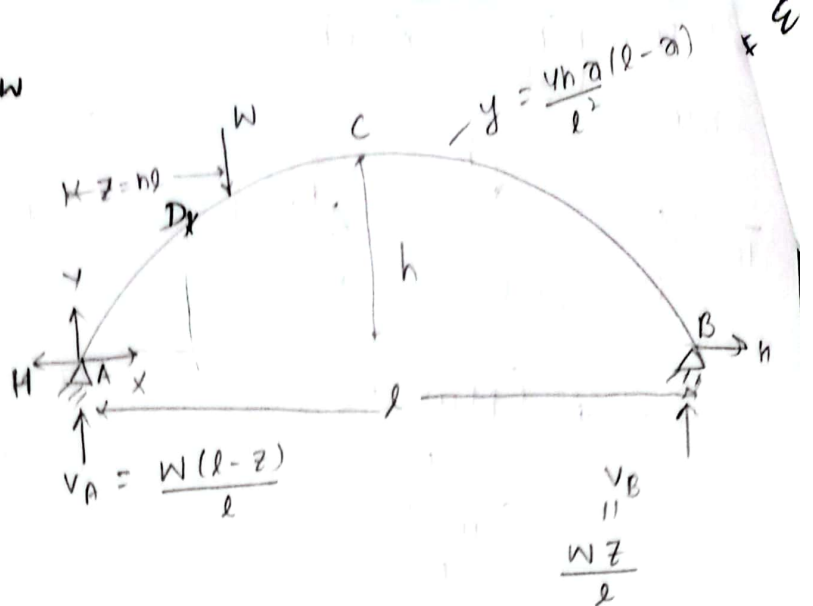
Thus,

$$H = - \frac{5}{8} \frac{W z}{h l^3} (l - z) (l^2 + l z - z^2)$$

Since, $z = n l$

$$\left[H = - \frac{5 W l}{8 h} (n - 2n^3 + n^4) \right] \leftarrow H z \text{ thrust due to point load } W \text{ at various location in arch}$$

$$\text{For, } n = 0.5 \rightarrow H = \frac{.25}{128} \frac{W l}{h}$$



* Effect of support yielding, Temperature changes and rib shortening

(I) Support yielding due to H₂ reaction (Elastic support case)

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$k_s \rightarrow$ Amount of support yielding due to unit of horizontal reaction
 unit: m/kN
 compatibility eqⁿ

$$\Delta_{10} + \delta_{11} H + k_s H = 0$$

$$\int \frac{My da}{EI_c} + H \int \frac{y^2 da}{EI_c} + k_s H = 0$$

$$H = - \frac{\int \frac{My da}{EI_c}}{\int \frac{y^2 da}{EI_c} + k_s}$$

(II) Support settlement initially without H.
 $\Delta_{10} + \delta_{11} H = \Delta$

$$H = \frac{\Delta - \int \frac{My da}{EI_c}}{\int \frac{y^2 da}{EI_c}}$$



(III) Temperature changes,

If temp $\uparrow$ by $\Delta t^\circ C$

$$\Delta_{10} + \delta_{11} H + \alpha l \cdot \Delta t = 0$$

(IV) Rib shortening under axial Force
 If arch is flat i.e. $l \uparrow$ & $h \downarrow$
 Amount of Rib shortening = $\frac{HL}{A_c E} = \Delta_{RS}$

$A_c \rightarrow$ c/s area at crown

Thru

$$\Delta_{10} + \delta_{11} H + \Delta_{RS} = 0$$

$\rightarrow$ Rib shortening

Considering all four effects,

$$\Delta_{10} + \delta_{11} H + k_s H + \Delta_{PS} + \alpha l \Delta t = \Delta$$

$$\left[\int \frac{M y d^2}{E I_c} \right] H \int \frac{y^2 d\theta}{E I_c} + k_s H + \frac{H l}{A_c E} + \alpha l \Delta t = \Delta$$

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$\propto$ Max^m +ve BM at location of pt. load.

$$\Rightarrow M_a = \underbrace{M_0}_{\text{Simply supported beam BMD for given loading in arch}} - \underbrace{H y}_{\text{Hz-thrust}}$$

Simply supported beam BMD for given loading in arch

Hz-thrust

Plastic Analysis.

• Frame Analysis

(1) Bernoulli Analysis

↳ only in member in which load is acting on its span

↳ 3 hinges necessary

(two at ends and one at intermediate position)

some may be mechanical hinges.

• For member with concentrated loads

↳ Number of possible mechanisms tried = number of concentrated loads.

• Load uniformly distributed:

↳ (i) If both ends of member are continuous (including fixed case) or simply supported

↳ Intermediate hinge develop at mid-span.

↳ (ii) If one end is continuous/fixed and other end is hinged/simply supported

↳ Intermediate hinge develops at 0.414L from hinged/simply supported ends.

(2) Sway Mechanism

↳ Due to lateral forces, frame may considerably sway.

↳ Two hinges at bottom and two hinges at the top of columns are necessary for this type of collapse.

↳ Hinges may mechanical hinges at bottom.

↳ work is done by plastic moments acting at plastic hinges.

(3) combined Mechanism:-

↳ Beam + Sway

↳ Real mechanism is one in which Internal work done is less and external work done is more, so that least collapse load or max^m plastic moment capacity required for existing.

↳ Sway on windward direction

#

→ collapse load = load factor × working load

$$\rightarrow \text{shape factor} = \frac{M_p}{M_y} = \frac{Z_p}{Z_y}$$

$$Z_p = \frac{A}{2} (y_t + y_c)$$

$$Z_y = \frac{I_{xx}}{y_{\max}}$$

→ For any span of beam for collapse,

↳ Natural (mechanical) or plastic hinges at support

↳ one plastic hinge in the span of beam

$$\text{Internal work (W}_i\text{)} = \sum (M_p)_i \times \theta_i$$

$$\text{External work (W}_e\text{)} = \text{Total load} \times \text{Avg. distance moved}$$

$$W_i = W_e \rightarrow \underline{M_p}$$

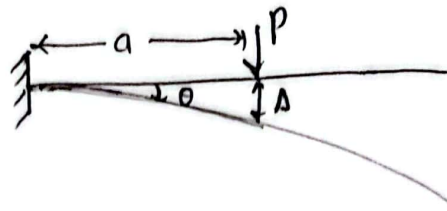
→ sudden change in section → hinge formed on weaker side.

In cantilever, deflection and rotations under the load 'P' acting at a distance 'a' from fixed end

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$$\Delta = \frac{Pa^3}{3EI}$$

$$\theta = \frac{Pa^2}{2EI}$$



Compatibility equation for force method:-

$$\delta_{11} R_1 + \delta_{12} R_2 + \Delta_{1S} = \Delta_1$$

$\Delta_{1S} \rightarrow$ when coordinate do not coincide with support settlement and rotation.

$$- \sum R_i \delta_i$$

$$= - (R_{a1} \times \Delta_{s1} + R_{b1} \times \Delta_{s2} + R_{\phi 1} \times \phi)$$

support settle or rotate गर्को ठाउँमा
unit load at 1 coordinate
ले गर्दा आएको
रखनु.

Three Moment equation:-

$$M_A \frac{L_1}{I_1} + 2M_B \left(\frac{L_1}{I_1} + \frac{L_2}{I_2} \right) + M_C \left(\frac{L_2}{I_2} \right) =$$

$$- \frac{6A_1 a_1}{I_1 L_1} - \frac{6A_2 a_2}{I_2 L_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$

Slope - Deflection equation:

$$M_{AB} = FEM_{AB} + \frac{2EI}{L} \left(2\theta_A + \theta_B - \frac{3\Delta}{L} \right)$$

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clw +ve

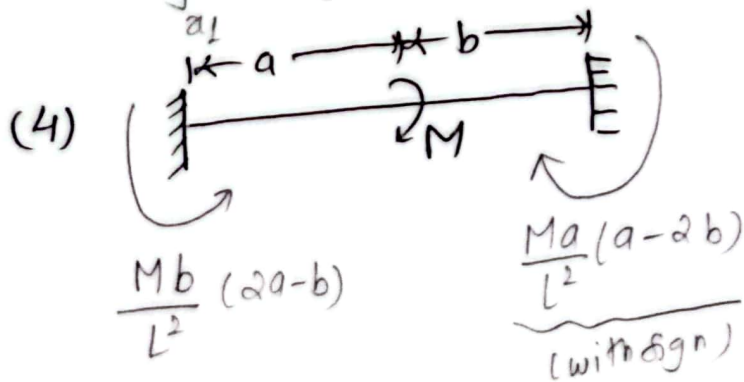
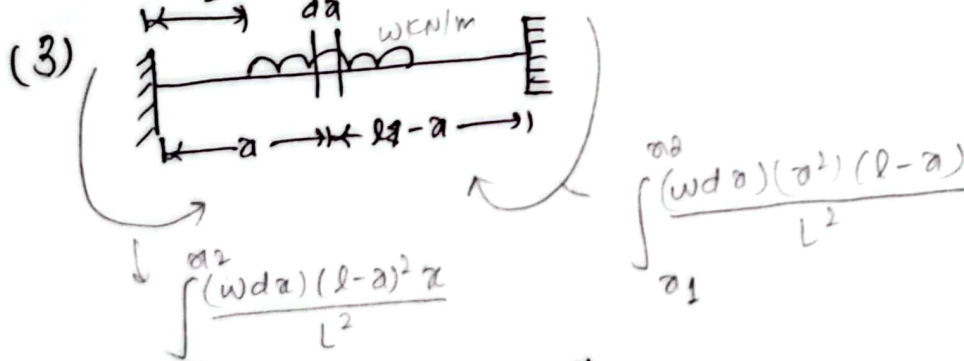
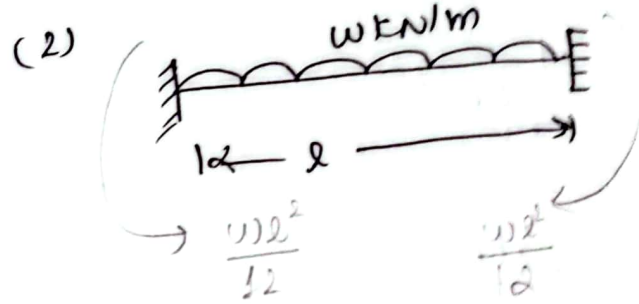
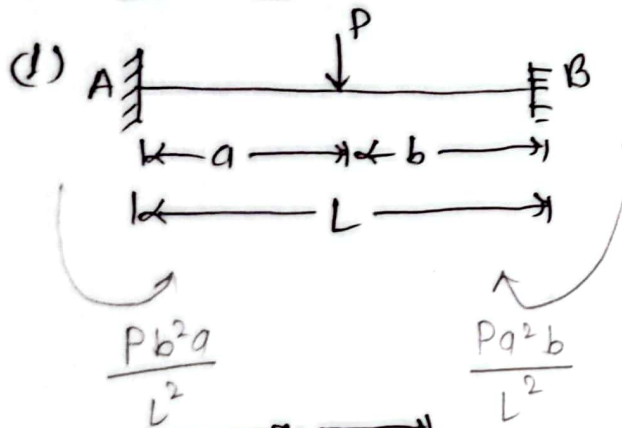
Acw -ve

clw +ve

Acw -ve

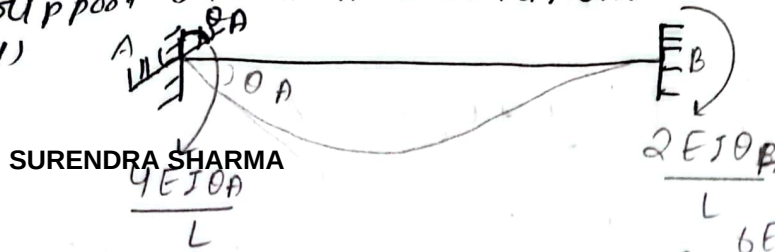
Left up right
down → +ve

FEM_{AB} → Both end considered fix.

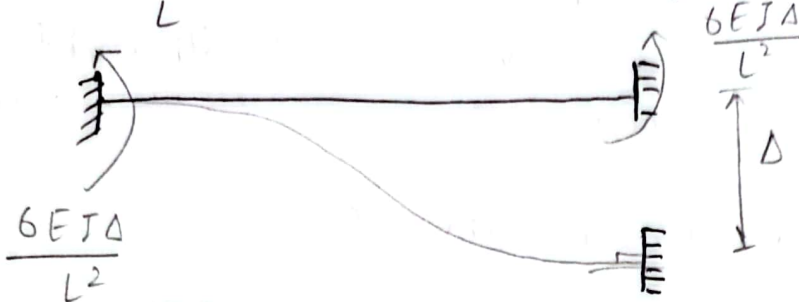


Support settlement & rotation:

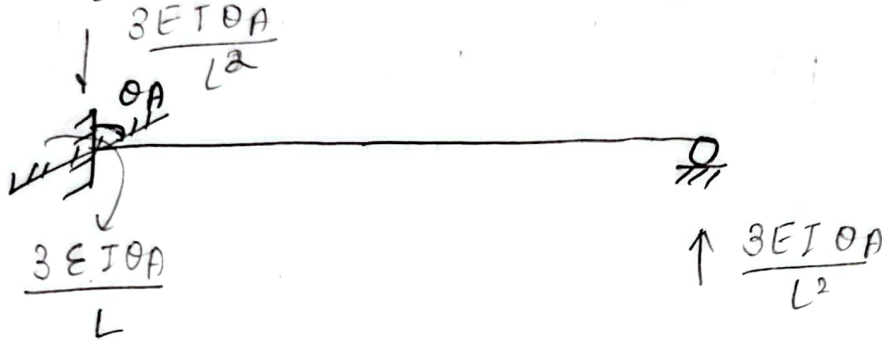
(1)



(2)



(3)



(4)

