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Subject: Structural Dynamics (Elective-I)

[Assignment]

[2079 Bhadra, Regular]

Q.1.(a) What is D'Alembert's principle and principle of conservation of energy? Explain.

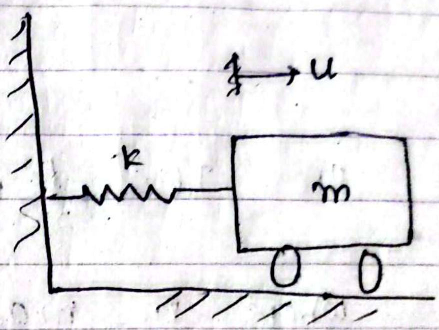
→ D'Alembert's principle states
"A system may be set in a state of dynamic equilibrium by adding to the external forces a fictitious force that is commonly known as the inertia force."

- It allows us to use equations of equilibrium in obtaining the equation of motion.

eg:

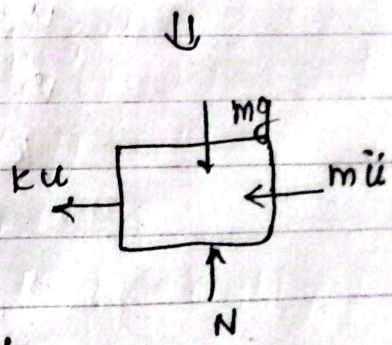
For given system,

By D'Alembert $[m\ddot{u} + ku = 0]$
[In u-direction)



which is equivalent to

Newton's law of motion $[-ku = m\ddot{u}]$



- Thus, D'Alembert's principle is used to establish dynamic equilibrium of the system.

Conservation of energy:

"The total energy of the system is always conserved."

" Energy can neither be created nor destroyed, but it can be transformed from one form to another."

i.e.

For undamped & conservative system.

Total Energy (TE) = Kinetic Energy (KE) + Strain/Potential energy (PE)

or, $TE = \frac{1}{2} m \dot{v}^2 + \frac{1}{2} * k \cdot v^2$

where
m = mass
$\dot{v}$ = velocity
v = displacement
k = spring constant.

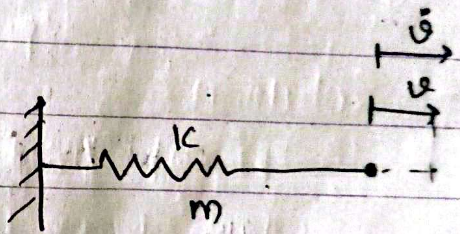
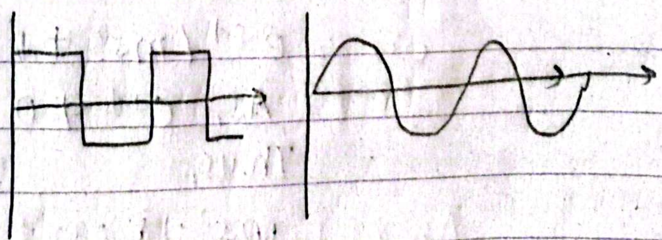


Fig:

(b) Differentiate harmonic load from periodic load with examples.

Harmonic load	Periodic load
(i) The magnitude of force acting on structure can be represented by a sine or cosine function of time.	(i) The load that repeats itself after a regular interval of time.
(ii) Eg: For undamped system $m\ddot{v} + kv = F_0 \sin \bar{\omega} t$ — ① 'OR' $m\ddot{v} + kv = F_0 \cos \bar{\omega} t$	(ii) Eg:- $F(t) = F(t+P)$ where, P is the period.
(iii) It doesn't need Fourier series to break it into sinusoidal form.	(iii) It may be broken into sinusoidal form by using Fourier series.
(iv) It forms smooth sine or cosine waves. Solution of ①: $v(t) = A\cos \omega t + B \sin \omega t + \frac{F_0/k}{1-\beta^2} \sin \bar{\omega} t$	(iv) It forms square, pulse, or sinusoidal waves. 
(v) Eg. Structure subjected to dynamic action of rotating machinery.	(v) Eg. Repeated impact loads (traffic loads on bridges)

(c) Derive equation for response of SDOF system for un-damped free vibration and explain natural frequency time period and amplitude of vibration

→ Soln.

Equation of motion for SDOF system is:

$$m\ddot{v}(t) + c\dot{v}(t) + kv(t) = p(t) \quad \text{— (I)}$$

For undamped free vibration,

$$m\ddot{v}(t) + kv(t) = 0 \quad \text{— (II)}$$

which is homogenous eqn of 2nd order.

General Solution of (II) is,

$$v(t) = e^{st} \quad \text{— (III)}$$
$$\dot{v}(t) = se^{st} \quad \text{— (IV)}$$
$$\ddot{v}(t) = s^2e^{st} \quad \text{— (V)}$$

Thus substituting in (II)

$$ms^2e^{st} + ke^{st} = 0$$

or $e^{st}(ms^2 + k) = 0$

As, $e^{st} \neq 0$

Thus,

$$ms^2 + k = 0$$

or, $s^2 = -\frac{k}{m}$

or, $s^2 = i^2\omega^2 \quad [\because \omega^2 = k/m]$ natural frequency

$\Rightarrow \boxed{s = \pm i\omega}$

$$v(t) = e^{\pm i\omega t}$$

The solution is rewritten as,

$$v(t) = A_1e^{i\omega t} + A_2e^{-i\omega t} \quad \text{— (VI)}$$

A_1 & A_2 → Arbitrary constant

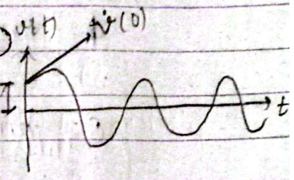
Using De-Moivre's theorem,

$$e^{i\omega t} = \cos\omega t + i\sin\omega t$$
$$e^{-i\omega t} = \cos\omega t - i\sin\omega t$$

Thus,

eqn (VI) becomes,

Final solution: $\boxed{v(t) = A\cos\omega t + B\sin\omega t}$ — (VII)



say, at, $t = 0$, $v(0)$ & $\dot{v}(0)$

$[v(0) = A]$.

$\left[\dot{v}(0) = B\omega \Rightarrow B = \frac{\dot{v}(0)}{\omega}\right]$

Thus,

General solution becomes,

$$\left[v(t) = v(0)\cos\omega t + \frac{\dot{v}(0)}{\omega}\sin\omega t\right] \quad \text{— (VIII)}$$

• Natural frequency, $\omega = \sqrt{\frac{k}{m}}$ rad/s. {Inherent frequency of system which depends only on structural property i.e. m & k}

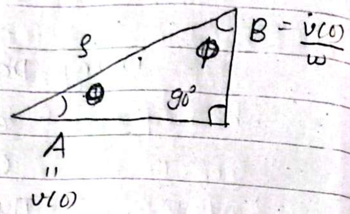
{k = stiffness of system
m = mass of system

• Time period (T); $\omega = \frac{2\pi}{T}$

$\Rightarrow \left[T = \frac{2\pi}{\omega}\right]$ → Time to complete one complete cycle

· Amplitude

↳ Maximum displacement of the system from its equilibrium position.



Thus,

$$\rho = \sqrt{A^2 + B^2}$$

$$= \sqrt{(v(0))^2 + \left(\frac{\dot{v}(0)}{\omega}\right)^2}$$

Also,

$$\phi = \tan^{-1}\left[\frac{v(0)}{\dot{v}(0)} \times \omega\right].$$

$$\theta = \tan^{-1}\left[\frac{\dot{v}(0)}{\omega\, v(0)}\right]$$

~~$A = \rho\cos\theta$ & $B = \rho\sin\theta$.~~

$A = \rho\sin\phi$ & $B = \rho\cos\phi$

Thus Eqⁿ (vii) becomes,

$$v(t) = \rho\sin(\omega t + \phi).$$

Q.2.(a) The SDOF system having viscous damping has a spring coefficient 500N/m, when the weight is displaced and released, the period of vibration is 2.0sec and ratio of successive amplitude is 4 to 1. Determine the amplitude of the motion, transmissibility ratio (TR) when a force $F(t) = 4\sin 4t$ is applied to the system.

→ Solⁿ:-

$K = 500\,N/m$

$T_D = 2\,sec$

$\frac{v_n}{v_{n+1}} = \frac{4}{1}$

$\rho = ?$

$TR = ?$

$F(t) = 4\sin 4t$

Thus,

$F_0 = 4N$

$\bar{\omega} = 4\,rad/s.$

Now,

→ Logarithmic decrement $(\delta) = \ln\left(\frac{v_n}{v_{n+1}}\right)$

$= \ln\left(\frac{4}{1}\right)$

$= 1.386$

→ Also $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$

~~Say, damping for system be ξ%.~~

or $1.386 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$ ⇒ solving, $\xi = 0.215$

For damped system,

$$\omega_D = \omega\sqrt{1-\xi^2}$$

or $\frac{2\pi}{T_D} = \frac{2\pi}{T}\sqrt{1-\xi^2}$

$\Rightarrow$ $T = T_D\sqrt{1-\xi^2}$

$= 2\sqrt{1-0.215^2}$

$= 1.953 sec.$

Natural frequency $\omega = \frac{2\pi}{T} = \frac{2\pi}{1.953} = 3.217$ rad/s.

Thus,

Frequency ratio, $\beta = \frac{\bar{\omega}}{\omega}$

$= \frac{4}{3.217} = 1.244$

$\Rightarrow$ Transmissibility ratio (TR) = $\sqrt{\frac{1+(2\xi\beta)^2}{(1-\beta^2)^2+(2\xi\beta)^2}}$

$= \sqrt{\frac{1+(2*0.215*1.244)^2}{(1-1.244^2)^2+(2*0.215*1.244)^2}}$

$[TR = \cancel{1.306} \approx 1.481]$.

$\Rightarrow$ Amplitude, $\delta = \frac{F_0}{k} * D$

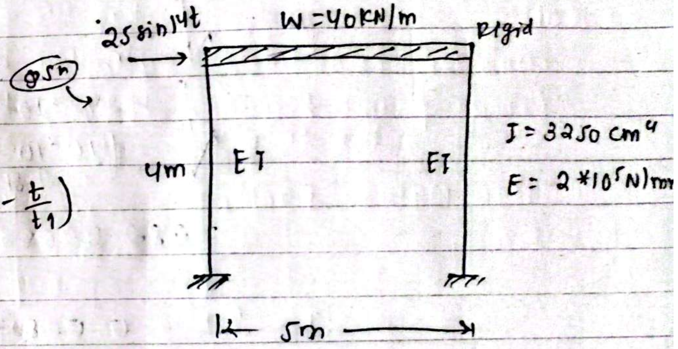
Dynamic amplification factor, $D = \frac{1}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}}$

$= \cancel{0.814} = 1.306$

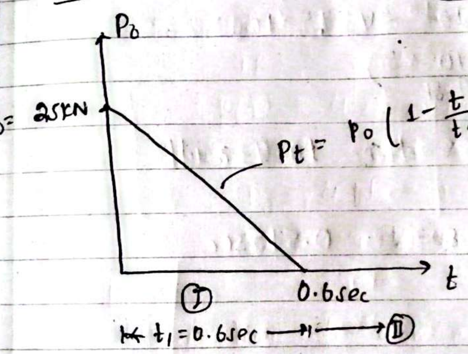
$\left[\delta = \frac{4}{500} * 1.306 = 10.45\,mm\right]$

Q 2.(b) For the steel frame as shown below which is subjected to a horizontal force applied at the girder level BC. The force decreases linearly from 25kN at time t=0 to zero at t=0.6sec. Determine (i) horizontal deflection at t=0.5sec.

(ii) the maximum horizontal deflection. Assume columns to be massless and girder to be rigid. Neglect damping.



→ Soln:



Thus, question refers a triangular impulse loading,

Solution ~~for~~ is:

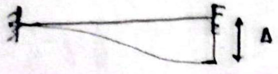
Phase (I).

$$v(t) = -\frac{P_0}{k}\cos\omega t + \frac{P_0}{kt_1\omega}\sin\omega t + \frac{P_0}{k}\left(1-\frac{t}{t_1}\right)$$

$$\dot{v}(t) = \frac{P_0}{k}\omega\sin\omega t + \frac{P_0}{kt_1}\cos\omega t - \frac{P_0}{kt_1}$$

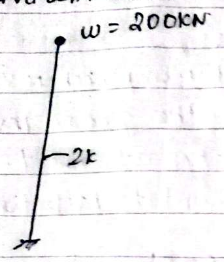
For maxima $\dot{v}(t) = 0$

$\Rightarrow$ $\omega t = 2.33$ rad.



$$k = \frac{12EI}{l^3}$$

Equivalent SDOF



$$2k = \frac{2 * 12 * EI}{l^3}$$

$$= \frac{24 * 2*10^5 * 3250 * (100)^4}{(4000)^3}$$

$k_{eq.} = \cancel{12.48*10^6\ N/mm}\ 2437.5\ KN/m$

$\cancel{[k_{eq} = 1.248*10^{10}\ N/m.]}$

$$w = \sqrt{\frac{K_{eq}}{m}}$$

$$= \sqrt{\frac{\cancel{1.248*10^{10}}\ 24375*1000}{\left(\frac{40*10^3*5}{9.81}\right)}}$$

$= \cancel{782.396\ rad/sec}\ 10.93\ rad/s$

$T = \frac{2\pi}{w} = \cancel{0.00803\ sec}\ 0.575\ sec.$

(i) $t = 0.5\ sec$ lies in (I)

so,

$$v(0.5) = -\frac{25*10^3}{2437.5*10^3} \cos(\overset{10.93}{\cancel{782.396}} * 0.5)$$

$$+ \frac{25*10^3}{2437.5*10^3 * 0.6 * 10.93} \sin(\overset{10.93}{\cancel{782.396}} * 0.5)$$

$$+ \frac{25*10^3}{2437.5*10^3}\left(1 - \frac{0.5}{0.6}\right)$$

$= \cancel{4.7766*10^{-4}\ mm} = -6.44\ mm$

(ii) Maximum Deflection.

~~at pt. t=0.6~~ $\frac{t_1}{T} = \frac{0.6}{0.575} = \overset{1.043}{\cancel{74.72}} \geq 0.371$

so lies in phase.

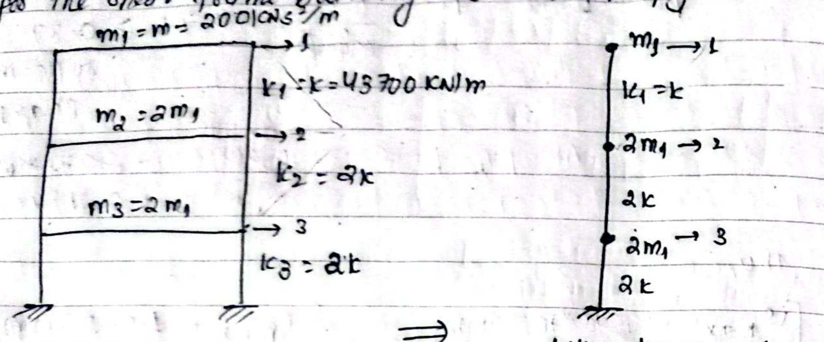
At pt, $t = 2.33 * \frac{0.575}{2\pi} = 0.213\ sec.$

Thus,

$[U_{max} = 15.57\ mm]$

~~Q3.(a) Determine the natural frequen~~

Q.3(a) Determine the natural frequencies and mode shapes for the shear frame building as shown in figure below.



Lumped mass system

→ Soln:-

Consider coordinate system as shown by arrow ($\rightarrow$)
(shear frame building → only translation and no rotation)

Mass matrix, $[m] = 200 \times 10^3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} Ns^2/m$

Stiffness matrix, $[k] = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_3 \end{bmatrix}$

$= 43700 \times 10^3 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 4 \end{bmatrix}$

Eigen value problem:

$$[[k] - \omega_n^2 [m]]\{\hat{v}_n\} = \{0\} \quad \text{— (I)}$$

Characteristic eqn:-

$$|[k] - \omega_n^2 [m]| = 0 \quad \text{— (II)}$$

or $\left| 43700 \times 10^3 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 4 \end{bmatrix} - \omega_n^2 \times 200 \times 10^3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right|$

or $43700 \times 10^3 \begin{vmatrix} (1-P) & -1 & 0 \\ -1 & (3-2P) & -2 \\ 0 & -2 & (4-2P) \end{vmatrix} = 0$

where $P = \dfrac{\omega_n^2 \times 200 \times 10^3}{43700 \times 10^3}$

Expanding:

$$(1-P)[(3-2P)(4-2P) - 4] + 1(-1)(4-2P) = 0$$

on solving

$P_1 = 0.255356 \Rightarrow \omega_1 = 7.47\,rad/s$
$P_2 = 1.355416 \Rightarrow \omega_2 = 17.209\,rad/s$
$P_3 = 2.88923 \Rightarrow \omega_3 = 25.126\,rad/s$

Natural frequency

For mode shapes

From (I)

$$\begin{bmatrix} (1-P) & -1 & 0 \\ -1 & (3-2P) & -2 \\ 0 & -2 & (4-2P) \end{bmatrix} \begin{Bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \hat{v}_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$[E_{01}^n] * \{\hat{v}_1\} + [E_{00}^n] * \{\hat{v}_{0n}\} = \{0\}$$

Normalizing $\{\hat{V}_1\}$ to unity

$$\{\hat{V}_{0n}\} = -[E_{00}^{n}]^{-1} \times [E_{01}^{n}]$$

$$\begin{Bmatrix} \hat{V}_2 \\ \hat{V}_3 \end{Bmatrix} = -\begin{bmatrix} (3-2p) & -2 \\ -2 & 4-2p \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \text{— (III)}$$

For 1st mode, $p_1 = 0.255356$, substituting in (III)

$$\begin{Bmatrix} \hat{\phi}_{11} \\ \hat{\phi}_{21} \\ \hat{\phi}_{31} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0.745 \\ 0.427 \end{Bmatrix}$$

For 2nd mode, $p_2 = 1.355416$

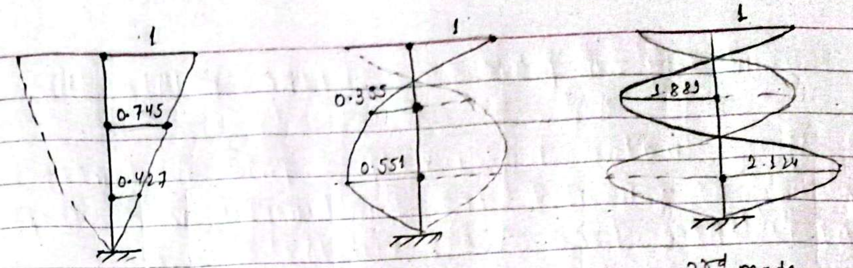
$$\begin{Bmatrix} \hat{\phi}_{12} \\ \hat{\phi}_{22} \\ \hat{\phi}_{32} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.355 \\ -0.551 \end{Bmatrix}$$

For 3rd mode, $p_3 = 2.88923$

$$\begin{Bmatrix} \hat{\phi}_{13} \\ \hat{\phi}_{23} \\ \hat{\phi}_{33} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1.889 \\ 2.124 \end{Bmatrix}$$

Thus, mode shape matrix,

$$[\phi] = \begin{bmatrix} 1 & 1 & 1 \\ 0.745 & -0.355 & -1.889 \\ 0.427 & -0.551 & 2.124 \end{bmatrix}$$



1st mode 2nd mode 3rd mode

Mode shapes

(b) Demonstrate numerically that the computed mode shape from above satisfies the orthogonality condition.

→ Soln:-

$$M_{12} = \{\phi_1\}^T [m] \{\phi_2\} = \begin{Bmatrix} 1 \\ 0.745 \\ 0.427 \end{Bmatrix}^T \times 200\times10^3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ -0.355 \\ -0.551 \end{Bmatrix}$$

$$= 99.2$$

$$= M_{21}$$

$$M_{13} = M_{31} = \{\phi_1\}^T [m] \{\phi_3\} = \begin{Bmatrix} 1 \\ 0.745 \\ 0.427 \end{Bmatrix} \times 200\times10^3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ -1.889 \\ 2.124 \end{Bmatrix}$$

$$= -142.8$$

$$M_{23} = M_{32} = \{\phi_2\}^T [m] \{\phi_3\} = 108.4$$

$$M_{11} = \{\phi_1\}^T [m] \{\phi_1\} = 494941.6$$

$$M_{22} = \{\phi_2\}^T [m] \{\phi_2\} = 371850.4$$

$$M_{33} = \{\phi_3\}^T [m] \{\phi_3\} = 3431878.8$$

comparing M_{12}, M_{13}, M_{23} with M_{11}, M_{22} & M_{33} they are negligible ≈ 0

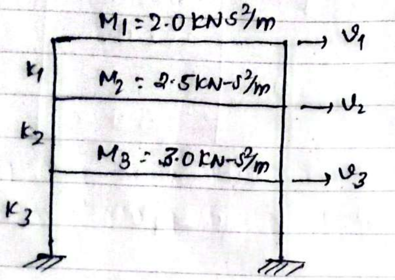
Thus,

This proves orthogonality condition (mass)

$\{\phi_m\}^T [m] \{\phi_n\} = 0$

Similarly can prove for stiffness orthogonality.

Q.4.(a) Using matrix holzer method, find fundamental period of vibration and fundamental mode shape of the three-storey shear building as given.



where,
$k_1 = 700 kN/m$
$k_2 = 1100 kN/m$
$k_3 = 1700 kN/m$

→ Sol^n:

$M_1 = 2000 N s^2/m$, $k_1 = 700 * 10^3 N/m$
$M_2 = 2500 Ns^2/m$, $k_2 = 1100 * 10^3 N/m$
$M_3 = 3000 Ns^2/m$, $k_3 = 1700 * 10^3 N/m$

Trial 1: Assume, $w^2 = 100 rad^2/s^2$

	v ($v_{i+1} = v_i - \Delta v$)	Δv ($= \mathcal{S}/K$)	$\mathcal{S}$ ($= \Sigma f_I$)	f_I ($= mv * w^2$)
①	1 (assume)			2000
		0.286	200	
②	0.714			178.5
		0.344	378.5	
③	0.370			111
		0.288	489.5	
ⓑ	0.082			

$v_b > 0$

As, it is in 3 mode thus, increase w^2.

Trial 2: $w^2 = 150 rad^2/s^2$

	v	Δv	$\mathcal{S}$	f_I
①	1 (assume)			300
		0.428	300	
②	0.572			214.5
		0.468	514.5	
③	0.104			46.8
		0.330	561.3	
ⓑ	-0.226			

$v_b < 0$

For Trial 3:

$$\frac{(\Delta w^2)_{1-2}}{(v_b)_{1-2}} = \frac{(\Delta w^2)_{2-3}}{(v_b)_{2-3}}$$

or $$\frac{150-100}{-0.226-0.082} = \frac{x-150}{0-(-0.226)}$$

$\Rightarrow (w^2)_3 = 113.31 \; rad^2/s^2$]

$\omega^2 = 113.35 \, rad^2/s^2$

	v	Δv	g	f_I
①	1 assuming			227
		0.324	227	
②	0.676			191.81
		0.381	418.81	
③	0.295			100.45
		0.305	519.26	
⑥	− 0.01			

$v_b \simeq 0$

Thus,

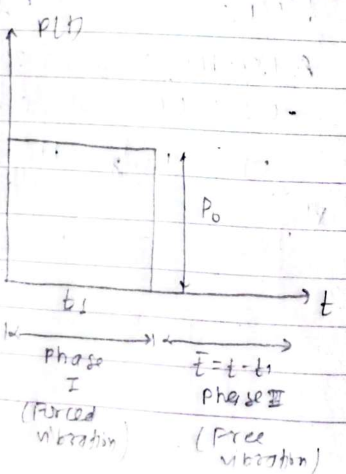
$\omega^2 = 113.5 \, rad^2/s^2$

Thus,

Fundamental period, $[\omega_1 = 10.65 \, rad/s]$.

Q.5 short notes:

(b) Rectangular impulse.



- Phase I (during impulse)

Eq^n of motion,

$$m\ddot{u}(t) + k\,u(t) = P_0$$

The particular solution is,

$$u_p = \frac{P_0}{k}.$$

Complementary solution is,

$$u_c = A\cos\omega t + B\sin\omega t$$

Total response

$$u(t) = u_p + u_c$$

$$u(t) = A\cos\omega t + B\sin\omega t + \frac{P_0}{k}$$

If initial condition is at rest i.e. $u(0) = 0$, $\dot{u}(0) = 0$.

$$0 = u(0) = A + \frac{P_0}{k}$$

$$\Rightarrow \left[A = -P_0/k\right]$$

$$\dot{u}(t) = -A\omega\sin\omega t + B\omega\cos\omega t.$$

$$\dot{u}(0) = 0 = B\omega$$

$$\Rightarrow [B = 0].$$

Thus,

$$\left[u(t) = \frac{P_0}{k}\left[1 - \cos\omega t\right]\right]$$

For maximum response in phase I,

$$\frac{d\,u(t)}{dt} = \dot{u}(t) = 0$$

or $\frac{P_0}{k} \times \omega \sin\omega t = 0$

or $\sin\omega t = 0$

or $\omega t = n\pi$ where $n = 0, \pm 1, \pm 2, \ldots$

For $n = 1$

$$\left[t = \frac{\pi}{\omega} = \frac{\pi}{2\pi} \times T = T/2\right]$$

Thus, for max^m response to occur in phase I,

$$t_1 \geq T/2$$

$$u_{max} = \frac{P_o}{k}\left[1 - \cos \omega * \frac{T}{2}\right] \quad \omega = 2\pi/T$$

$$= \frac{P_o}{k}\left[1-(-1)\right]$$

$$\boxed{u_{max} = \frac{2P_o}{k}}$$

$$\boxed{D = \frac{u_{max}}{P_o/k} = 2}$$

Phase II (after impulse)

$$[\bar{t} = t - t_1 \geq 0]$$

The undamped free vibration response in phase II is,

$$u(\bar{t}) = u(t_1)\cos\omega\bar{t} + \frac{\dot{u}(t_1)}{\omega}\sin\omega\bar{t}$$

Amplitude

$$\rho = \sqrt{[u(t_1)]^2 + \left(\frac{\dot{u}(t_1)}{\omega}\right)^2}$$

$$u(t_1) = \frac{P_o}{k}[1-\cos\omega t_1]$$

$$\dot{u}(t_1) = \frac{P_o}{k}\,\omega\sin\omega t_1$$

$$\rho = \frac{P_o}{k}\sqrt{(1-\cos\omega t_1)^2 + (\sin\omega t_1)^2}$$

$$\text{or} \quad \left[\rho = \frac{P_o}{k}\sqrt{2(1-\cos\omega t_1)}\right]$$

(d) Uncoupled equation of motion:

For undamped forced vibration:

$$[m]\{\ddot{v}\} + [k]\{v\} = \{P(t)\} \quad \text{—(1)}$$

Also, $\{v\} = [\Phi]\{Y\}$ ← Amplitude vector

$[\Phi]$: shape matrix

$\{v\}$: Displacement vector

$$\{\ddot{v}\} = [\Phi]\{\ddot{Y}\}$$

Substituting in (1),

$$\text{or} \quad [m][\Phi]\{\ddot{Y}\} + [k][\Phi]\{Y\} = \{P(t)\}$$

Premultiply by $\{\Phi_n\}^T$ both sides,

$$\text{or} \quad \{\Phi_n\}^T[m][\Phi]\{\ddot{Y}\} + \{\Phi_n\}^T[k][\Phi]\{Y\} = \{\Phi_n\}^T\{P(t)\}$$

Using orthogonality condition,

$$\text{or} \quad \underbrace{\{\Phi_n\}^T[m]\{\Phi_n\}}\ddot{Y}_n + \underbrace{\{\Phi_n\}^T[k]\{\Phi_n\}}Y_n = \underbrace{\{\Phi_n\}^T\{P(t)\}}$$

$$\text{or} \quad \left[M_n\ddot{Y}_n + K_n Y_n = P_n(t)\right]$$ ← uncoupled eq^n for undamped forced vibration

where,

M_n, K_n & $P_n(t)$ are Normal co-ordinates for generalized mass, stiffness and load for mode 'n'.

Similarly for damped ~~eqn~~ motion,

$$[m]\{\ddot{v}\} + [c]\{\dot{v}\} + [k]\{v\} = \{P(t)\}$$ ← coupled eq

$$\left[M_n\ddot{Y}_n + C_n\dot{Y}_n + K_n Y_n = P_n(t)\right]$$ ← uncoupled equation.

CamScanner

[2080 Bhadra] (Regular)

078BIE179

Q.1:(a) Explain with example the types of vibrations

→ Types of vibration:

(1) • Free → when a system is displaced from its equilibrium position and then allowed to vibrate without any external force acting on it (natural frequency)
eg: A structure vibrating after an earthquake shock (after ground motion stops)

• Forced → when a system is subjected to a continuous external time-varying force.
→ system vibrates at the frequency of the applied force.
eg. Bridges vibrating under moving vehicles

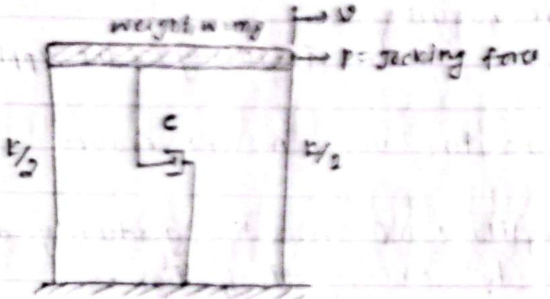
(2) • Damped → when amplitude of vibration gradually decrease over time due to energy dissipation.
(oscillation die out with time)
eg: Car suspension system (shock absorbers)

• Undamped → When there is no energy dissipation in the system, the amplitude remains constant, and vibration continues indefinitely. (Ideal case)
eg: Ideal pendulum without air resistance.

(3) • Longitudinal → Motion along axis (|| to length) :[eg: rod in te
• Transverse → vibration ⊥ to axis (eg. guitar string)
• Torsional → Angular oscillations (eg. rotating shaft)

(4) • Deterministic → known $p(t)$
• Random → stochastic (eg. wind, traffic)

Q.1.6) Assume that the mass and stiffness of fig. are as follows:
$m = 2$ kips·sec²/in, $k = 40$ kips/in. If the system is set into the free vibration with the initial condition $v(0) = 1.0$ and $\dot{v}(0) = 6$ in/sec, determine the displacement and velocity at $t = 1.2$ sec, assuming: (i) $c = 0$ (undamped system) (ii). $c = 2.8$ kip·sec/in.



→ Soln:

(i) For undamped system

General solution,

$u(t) = A\cos\omega t + B\sin\omega t$ —(I)

For $t=0$, $u(0) = 1.0$ in

$\dot{u}(0) = 6$ in/sec

$\dot{u}(t) = -A\omega\sin\omega t + B\omega\cos\omega t$ —(II)

[078BCE178]

$u(0) = A$

$\rightarrow [A = u(0) = 1.0]$

$\dot{u}(0) = 6 = B\omega$

$\rightarrow \left[B = \frac{6}{\omega}\right]$

$m = 2$ kips sec²/in

$k = 40$ kips/in

$\omega = \sqrt{k/m} = \sqrt{40/2} = 4.472$ rad/s.

$\left[B = \frac{6}{4.472} = 1.342\right]$

From (I),

$u(t) = 1.0\cos(\omega t) + 1.342\sin(\omega t)$ —(III)

For $t = 1.2$ sec,

$u(1.2) = 1.0\cos(4.472 * 1.2) + 1.342\sin(4.472 * 1.2)$

$[u(1.2) = -0.457 \text{ in}]$

$\dot{u}(t) = -1 * 4.472\sin(4.472t) + 1.342 * 4.472 * \cos(4.472 * t)$ —(IV)

For $t = 1.2$ sec.

$[\dot{u}(1.2) = 7.2 \text{ in/sec}]$

(ii) For damped system.

critical damping coeff, $c_{cr} = 2m\omega$

$= 2 * 2 * 4.472$

$= 17.89$ kips·sec/in.

$c = 2.8$ kips sec/in $< c_{cr}$.

(underdamped).

Solution,

$$u(t) = e^{-\xi\omega t}\left[u(0)\cos\omega_D t + \left(\frac{\dot{u}(0) + \xi\omega u(0)}{\omega_D}\right)\sin\omega_D t\right]$$

$$\xi = \frac{c}{2m\omega} = \frac{2.8}{17.89} = 0.1565$$

$$\omega_D = \omega\sqrt{1-\xi^2} = 4.472\sqrt{1-(0.1565)^2} = 4.417 \text{ rad/s.}$$

$u(0) = 1.0$ in

$\dot{u}(0) = 6.0$ in/s.

At $t = 1.2$ sec

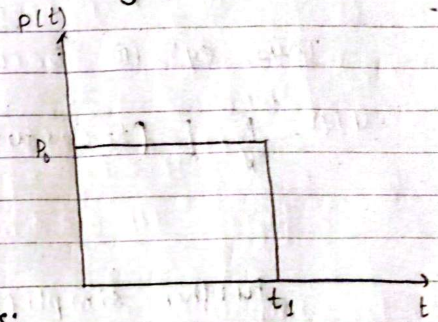
$$u(1.2) = e^{-0.1565 * 4.472 * 1.2}\left[1 * \cancel{\cos 4.417} \cos(4.417 * 1.2) + \left(\frac{6 + 0.1565 * 4.472 * 1}{4.417}\right)\sin(4.417 * 1.2)\right]$$

$$= 0.4318\left[1 * 0.555 + 1.5168 * (-0.832)\right]$$

$$= -0.305 \text{ in}$$

$[\dot{u}(t) = 3.405 \text{ in/sec}]$.

Q.2(a) For a SDOF system is subjected to step load of amplitude P_0 and duration t_1 sec; as shown below. Determine the response of undamped system during forced and free vibration phases.



→ Equation:

$$m\ddot{u} + ku = p(t) = \begin{cases} P_0 & t \le t_1 \\ 0 & t \ge t_1 \end{cases}$$

with at rest initial conditions:
$u(0) = \dot{u}(0) = 0$.

- Forced vibration phase: (During this phase, system is subjected to step force)

$$u(t) = \frac{P_0}{k}(1 - \cos\omega t)$$

$$u(t) = \frac{P_0}{k}\left[1 - \cos\left(\frac{2\pi}{T} * t\right)\right] \quad \text{for } t \le t_1 \quad \text{—(1)}$$

- Free vibration phase

After the force ends, at t_1, the system undergoes free vibration as,

$$u(t) = u(t_1)\cos\omega(\bar{t}) + \frac{\dot{u}(t_1)}{\omega}\sin(\omega\bar{t}), \quad t \ge t_1 \quad \text{—(ii)}$$

where $\bar{t} = t - t_1$

This free vibration is initiated by displacement and velocity of the mass at $t = t_1$, determined from eqn (1) as,

$$u(t_1) = \frac{P_0}{k}\left[1-\cos(\omega t_1)\right]$$

$$\dot{u}(t_1) = \frac{P_0}{k} * \omega \sin(\omega t_1)$$

Thus, eq^n (ii) becomes,

$$u(t) = \frac{P_0}{k}\left[\{1-\cos(\omega t_1)\}\cos(\omega \bar{t}) + \sin\omega t_1 \sin\omega \bar{t}\right]$$

for, $t \geq t_1$ — (iii)

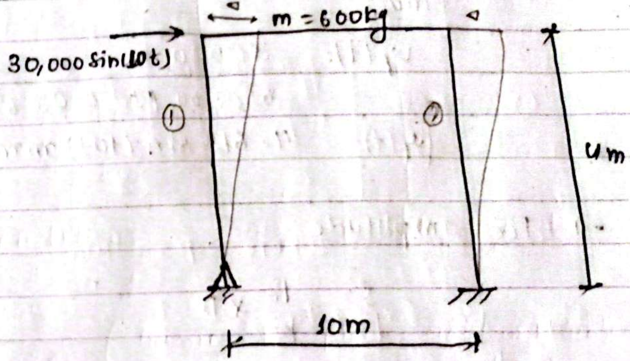
Further simplified using trigonometric identities,

$$u(t) = \frac{P_0}{k}\left[\cos(\omega \bar{t}) - \cos\omega(t_1 + t - t_1)\right]$$

$$\left[u(t) = \frac{P_0}{k}\left[\cos(\omega \bar{t}) - \cos(\omega t)\right] \quad , t \geq t_1\right]$$ w.

(078BCE178)

[Q. 2(b)] A steel frame shown in fig. support a rotating machine, which exert a horizontal force at girder level of $30{,}000 * \sin(10t)$ N. Take damping as 4% critical damping, moment of inertia of a column $14 \times 10^{-5}\ m^4$, $E = 2*10^5\ N/mm^2$. Determine steady state motion and amplitude of vibration.



→ Solution,

column,

$I = 14 * 10^{-5} m^4$

$E = 2*10^5 N/mm^2$

$E = 2*10^{11} N/m^2$

$$k_1 = \frac{3EI}{L^3}$$

$$k_2 = \frac{12EI}{L^3}$$

→ k: $k_{eq} = k_1 + k_2 = \frac{15EI}{L^3} = \frac{15 * 2*10^{11} * 14*10^{-5}}{4^3}$
(|| system)

$= 6562500$ N/m.

→ $\xi = 4\%$

→ $c = 2m\omega_n\xi = 2 * 600 * \underbrace{\sqrt{\frac{6562500}{600}}}_{104.58\ rad/s} * 0.04 = 5019.96$ N-s/m

→ Steady state responce,

$$u_p(t) = \frac{P_0}{k(1-\beta^2)} \sin(\bar{\omega} t)$$

$\bar{\omega} = 10$ rad/s

Thus

$$\beta = \frac{\bar{\omega}}{\omega} = \frac{10}{104.58} = 0.0956$$

Thus

$$u_p(t) = \frac{30000}{6562500\,(1-0.0956^2)} \sin(10t)$$

$$\left[u_p(t) = 4.619 \sin(10t)\ \text{mm}\right] \text{ans.}$$

→ For amplitude,

$$\rho = \frac{P_0}{k} * \hat{D}$$

$$D = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}$$

$$= \frac{1}{\sqrt{(1-0.0956^2)^2 + (2*0.04*0.0956)^2}}$$

$$= 1.0092$$

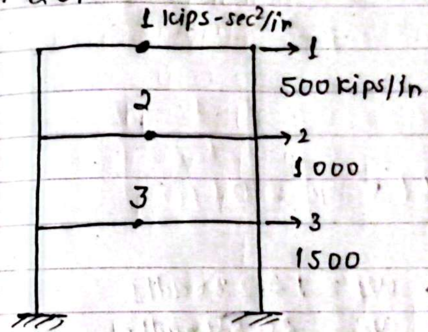
$$\left[\rho = \frac{30000}{6562500} * 1.0092 = 4.6135\ \text{mm}\right]$$

3.(a) A three storied massless frame is idealized in such a way that the lumped mass in the third, second and first floors are respectively 1, 2, and 3 kips-sec²/in. Column stiffness in the third, second and first stories are respectively 500, 1000 and 1500 kips/in. The columns are clamped at the bottom. Obtain the

(i) Vibration frequencies

(ii) Determine and draw vibration mode shapes.

→ Sol^n:



Columns are clamed at the bottom ⇒ base of the coumn is fixed (no translation or rotation)

Now, (coordinates are assumed as shown by →)

$$[m] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{kips-sec}^2/\text{in}$$

$$[k] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = 500 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix} \text{kips/in}$$

(i) Characteristic equation:

$$|[K] - \omega_n^2 [m]| = 0$$

or, $$\left| 500 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix} - \omega_n^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right| = 0$$

or, $$500 \begin{vmatrix} (1-P) & -1 & 0 \\ -1 & (3-2P) & -2 \\ 0 & -2 & (5-3P) \end{vmatrix} = 0$$

where $P = \dfrac{\omega_n^2}{500}$

On Expanding,

or, $(1-P)[(3-2P)(5-3P) - 4] - 1(5-3P) = 0$

On solving

$P_1 = 0.29912 \rightarrow \omega_1 = 12.23$ rad/s

$P_2 = 1.30422 \rightarrow \omega_2 = 25.54$ rad/s

$P_3 = 2.56334 \rightarrow \omega_3 = 35.80$ rad/s

vibration frequencies.

(ii) Eigen value problem:

$$[[K] - \omega_n^2 [m]]\{\hat{v}_n\} = \{0\}$$

or, $$\begin{bmatrix} (1-P) & -1 & 0 \\ -1 & (3-2P) & -2 \\ 0 & -2 & (5-3P) \end{bmatrix} \begin{Bmatrix} \hat{v}_{1n} \\ \hat{v}_{2n} \\ \hat{v}_{3n} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

floor

mode

(078BCE178)

where, $\{E_{01}^n\} = \begin{Bmatrix} -1 \\ 0 \end{Bmatrix}$

$E_{00}^n = \begin{bmatrix} (3-2P) & -2 \\ -2 & (5-3P) \end{bmatrix}$

$\hat{v}_{01}^n = \begin{Bmatrix} \hat{v}_{2n} \\ \hat{v}_{1n} \end{Bmatrix}$

Thus,

$$\{E_{01}^n\} * \hat{v}_{1n} + [E_{00}^n] * \{\hat{v}_{01}^n\} = 0$$

Let, $\hat{v}_{1n} = 1$ ← Top floor displacement normalized.

Then,

$$\{\hat{v}_{01}\} = -[E_{00}^n]^{-1}\{E_{01}^n\}$$

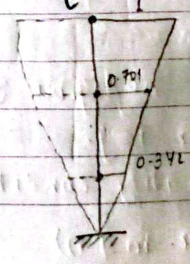
or, $$\begin{Bmatrix} \hat{v}_{2n} \\ \hat{v}_{3n} \end{Bmatrix} = - \begin{bmatrix} (3-2P) & -2 \\ -2 & (5-3P) \end{bmatrix}^{-1} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} \quad \text{—(1)}$$

From (1)

For 1st mode

$P_1 = 0.29912$

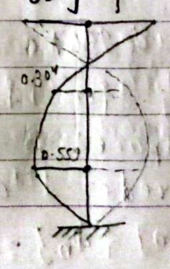
$$\begin{Bmatrix} \hat{v}_{11} \\ \hat{v}_{21} \\ \hat{v}_{31} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0.701 \\ 0.342 \end{Bmatrix}$$



For 2nd mode

$P_2 = 1.30422$

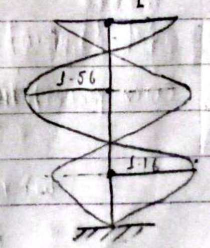
$$\begin{Bmatrix} \hat{v}_{12} \\ \hat{v}_{22} \\ \hat{v}_{32} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.304 \\ -0.559 \end{Bmatrix}$$



For 3rd mode

$P_3 = 2.56334$

$$\begin{Bmatrix} \hat{v}_{13} \\ \hat{v}_{23} \\ \hat{v}_{33} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1.56 \\ 1.16 \end{Bmatrix}$$



3. (b) Demonstrate above problem (9a) satisfy I orthogonality condition.

→ Sol^n:-

$M_{12} = M_{21} = \{\phi_1\}^T [m] \{\phi_2\}$

$$= \begin{Bmatrix} 1 \\ 0.701 \\ 0.342 \end{Bmatrix}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ -0.304 \\ -0.559 \end{Bmatrix}$$

$= 3.33 \times 10^{-3}$

$M_{13} = M_{31} =$
$= \{\phi_1\}^T [m] \{\phi_3\} = \begin{Bmatrix} 1 \\ 0.701 \\ 0.342 \end{Bmatrix}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ -1.563 \\ 1.162 \end{Bmatrix}$

$= 8.86 \times 10^{-4}$

$M_{23} = M_{32} = \{\phi_2\}^T [m] \{\phi_3\} = 0.0121$

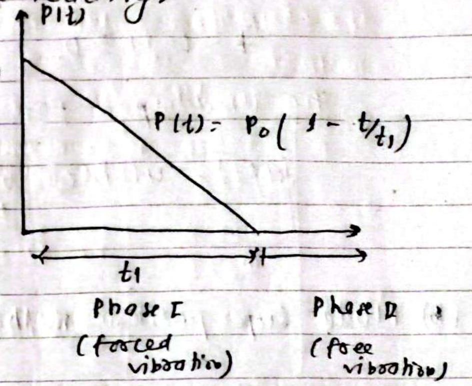
$M_{11} = \{\phi_1\}^T [m] \{\phi_1\} = 1.333$

$M_{22} = \{\phi_2\}^T [m] \{\phi_2\} = 2.1122$

$M_{33} = \{\phi_3\}^T [m] \{\phi_3\} = 9.9367$

Comparitivity M_{ij} for $i \neq j$ are negligible $\simeq 0$ compared to M_{ij} for $i = j$.
which proveds 1st orthogonality condition.
i.e. $\{\phi_m\}^T [m] \{\phi_n\} = 0$ (for $m \neq n$)

9.5 (a) Response to triangular impulsive loading:
(Ramp loading)



Here,
$p(t) = P_o \left(1 - \frac{t}{t_1}\right)$

Phase I : $0 \le t \le t_1$

$u_p = \frac{P_o}{K}\left(1 - \frac{t}{t_1}\right)$

$u_c = A\cos\omega t + B\sin\omega t$

Thus,
total response,
$u(t) = u_c + u_p$
$u(t) = A\cos\omega t + B\sin\omega t + \frac{P_o}{k}\left(1 - \frac{t}{t_1}\right)$.

Initial condition at rest and origin.
$u(0) = 0$ & $\dot{u}(0) = 0$

so, $u(0) = 0 = A + P_o/k \Rightarrow \left[A = -\frac{P_o}{K}\right]$

$\dot{u}(t) = -A\omega\sin\omega t + B\omega\cos\omega t - \frac{P_o}{kt_1}$

$\dot{u}(0) = 0 = B\omega - \frac{P_o}{kt_1} \Rightarrow \left[B = \frac{P_o}{k\omega t_1}\right]$

Thus, total solution

$u(t) = \frac{P_o}{K}\left[\frac{1}{\omega t_1}\sin\omega t - \cos\omega t + \left(1 - \frac{t}{t_1}\right)\right]$. —(1)

Phase II: $t \geq t_1$ (after impulse)

$\bar{t} = t - t_1 \geq 0$

The undamped free vibration response is given by

$$\left[u(\bar{t}) = u(t_1)\cos\omega\bar{t} + \frac{\dot{u}(t_1)}{\omega}\sin\omega\bar{t}\right] \quad \text{--- (ii)}$$

(b) Mode superposition method:

- The equation of motion for undamped system is:

$[m]\{\ddot{v}(t)\} + [k]\{v(t)\} = \{P(t)\}$ --- (i)

Introducing normal coordinates:

$\{v(t)\} = [\Phi]\{y\}$

$\{\ddot{v}(t)\} = [\Phi]\{\ddot{y}\}$

Thus,

or, $[m][\Phi]\{\ddot{y}\} + [k][\Phi]\{y\} = \{P(t)\}$

Premultiply by $\{\Phi_n\}^T$

or, $\{\Phi_n\}^T[m][\Phi]\{\ddot{y}\} + \{\Phi_n\}^T[k][\Phi]\{y\} = \{\Phi_n\}^T\{P(t)\}$

Using orthogonality property

or, $\underline{\{\Phi_n\}^T[m]\{\Phi_n\}}\,\ddot{y}_n + \underline{\{\Phi_n\}^T[k]\{\Phi_n\}}\,y_n = \underline{\{\Phi_n\}^T\{P(t)\}}$

or, $[\underline{M_n}\ddot{y}_n + \underline{K_n}y_n = \underline{P_n(t)}]$ --- (ii)

where

M_n, K_n & $P_n(t)$ are Generalized mass, stiffness and load.

[078BCE178]

Eq^n (i) → coupled equation.

Eq^n (ii) → uncoupled equation of motion for undamped system and represent the SDOF equation of motion for n^{th} mode.

Hence, dynamic response can be obtained by solving separately for each response.

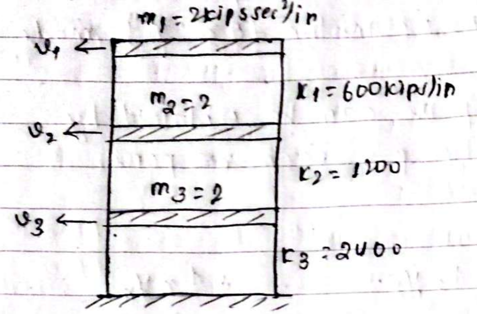
Total response,

$v(t) = \Phi_1 y_1 + \Phi_2 y_2 + \cdots + \Phi_n y_n$

$$\left[v(t) = \sum_{i=1}^{n} \Phi_i y_i\right]$$

This method is known as mode superposition method.

(Q) Find Fundamental vibration mode and frequency for the structure as shown in figure 2 using Stadola method.



→ Sol^n:

We have,

$$[m] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{kips.sec}^2/\text{in}$$

$$[k] = 600 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 6 \end{bmatrix} \text{kips/in.}$$

Flexibility matrix, $[a] = [k]^{-1}$

$$= \frac{1}{2400} \begin{bmatrix} 7 & 3 & 1 \\ 3 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[D] = [a][m] = \frac{1}{2400} \begin{bmatrix} 14 & 6 & 2 \\ 6 & 6 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

Trial 1

$$\{v_1^{(0)}\} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \text{ (assume)}.$$

$$\frac{1}{\omega^2} \{v_1^{(1)}\} = \{\bar{v}_1^{(1)}\} = [D]\{v_1^{(0)}\} \quad \text{—(1)}$$

Thus,

$\{v_1^{(0)}\}$	$\{\bar{v}_1^{(1)}\}$
1	22/2400
1	14/2400
1	6/2400

Trial 2:

$\{v_1^{(1)}\}$	$\{\bar{v}_1^{(2)}\}$
1	18.364
0.636	10.364
0.273	3.818

[Normalizing $\{\bar{v}_1^{(1)}\}$]

Trial 3:

$\{v_1^{(2)}\}$	$\{\bar{v}_1^{(3)}\}$
1	17.802
0.564	9.802
0.208	3.544

Trial 4:

$\{v_1^{(3)}\}$	$\{\bar{v}_1^{(4)}\}$
1	17.702
0.551	9.702
0.199	3.499

Stopping here.

Thus, $\{v_1^{(4)}\} = \begin{Bmatrix} 1 \\ 0.548 \\ 0.198 \end{Bmatrix} = \{\Phi_1\}$

(first mode shape)

From equation ①

For higher iteration

$$\omega^2 = \frac{\{v_1^{(k)}\}_{max}}{\{\bar{v}_1^{(k)}\}_{max}}$$

Taking maximum value of 4th iteration

$$\omega^2 = \frac{1}{17.702/2400} = 136.578 \text{ rad}^2/\text{sec}^2$$

Thus, $[\omega_1 = 11.644 \text{ rad/s}]$

Fundamental frequency.

- Check ω.

Range:

$$(\omega_1)^2_{min} = \frac{v_{11}^{(0)}}{\bar{v}_{11}^{(0)}} = \frac{1}{22/2400} = 109.09 \text{ rad}^2/\text{sec}^2$$

$$(\omega_1)^2_{max} = \frac{v_{31}^{(0)}}{\bar{v}_{31}^{(0)}} = \frac{1}{6/2400} = 400 \text{ rad}^2/\text{sec}^2$$

ω_1^2 from iteration within 109.09 & 400.

Approximate solution,

$$\omega^2 = \frac{\{\bar{v}_1\}^T [m] \{v_1(0)\}}{\{\bar{v}_1\}^T [m] \{\bar{v}_1\}}$$

$$= \frac{\frac{1}{2400}[22 \;\; 14 \;\; 6] \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}}{\frac{1}{2400}[22 \;\; 14 \;\; 6] \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} 22/2400 \\ 14/2400 \\ 6/2400 \end{Bmatrix}}$$

(078BCE178)

$$= \frac{0.035}{179/720000}$$

$$= 140.782$$

$\Rightarrow [\omega = 11.865 \text{ rad/s}]$.

OK

[2081 Bhadra] (Regular)

(078BCE178)

Q.1.(a) Describe prescribed loading and its types with suitable examples and also list the essential characteristics of the dynamic problem. Also, illustrate the essential characteristics of dynamic problems.

→ If the time variation of loading is fully known even though it may be highly oscillatory, irregular in character is called prescribed loadings.
Characteristics like magnitude, direction and location are known beforehand.

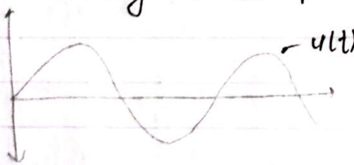
→ Types:

(i) Periodic loading

→ Load that repeats its values at regular interval (one cycle)

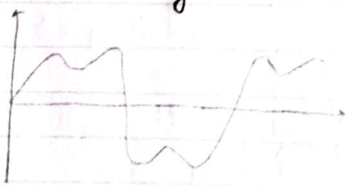
example:

(i) Harmonic loading → simple harmonic motion



$u(t) = A \sin(\omega t + \phi)$

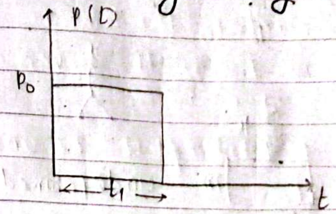
(ii) Non-harmonic loading



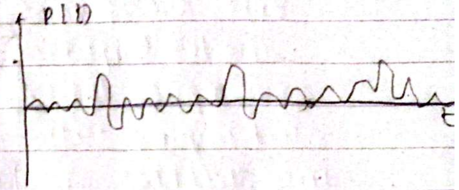
(ii) Non-periodic loading

- Load that do not repeat itself at regular interval.

eg: Transient loading, → load that comes & goes quickly
Impulsive loading → for short duration
Arbitrary loading → random (eg. earthquake)



eg. Rectangular impulse



eg. Arbitrary loading

• Essential characteristics of Dynamic problem:

① Both loading and response vary with time, so dynamic problem does not have a single solution. The analyst must establish a succession of solution corresponding to all time of interest in the response history.

② Due to friction, material internal resistance and other damping force amplitude reduces over time.

③ Dynamic response of problem depends not only on load but upon the internal forces which oppose ~~best~~ acceleration producing them.

④ Each structure vibrate in specific pattern (mode shap and specific frequency.

Further: ⇒ Governing equations are Differential equations

⇒ Time-Dependent loading

⇒ Inertia effects (F = ma)

⇒ Damping effects (dissipating energy

⇒ Full response (Displacement, velocity and acceleration) → time depen

⇒ Requires Boundary and initial conditions.

⇒ Every structure has natural frequencies and mode shape, Q.1.b) Assuming a massless simply supported beam, having length (L) and flexural rigidity (EI), which supports the mass (m) at its mid span from a spring of stiffness (k) at a mid point of beam as shown below. Determine the natural cyclic frequency of the suspended mass.

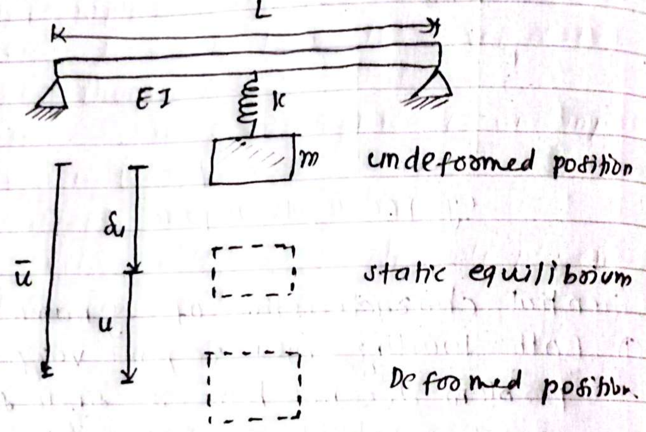


fig (i)

→ Soln:

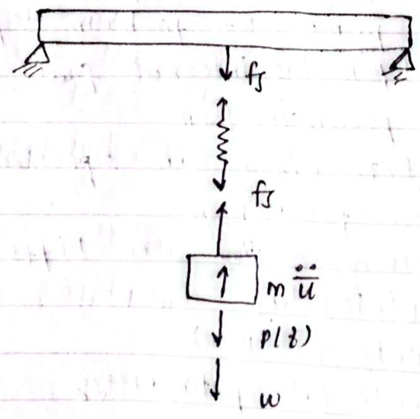


fig (ii)

(078BCE198)

1. write the equation of motion:

Equilibrium of forces in fig (ii) gives

$$m\ddot{\bar{u}} + f_s = w + p(t) \quad \text{— (a)}$$

where,

$$f_s = k_e \bar{u} \quad \text{— (b)}$$

The equation of motion is:

$$m\ddot{\bar{u}} + k_e \bar{u} = w + p(t) \quad \text{— (c)}$$

2. Determine the effective stiffness:

$$f_s = k_e \bar{u}$$

where,

$$\bar{u} = \delta_{spring} + \delta_{beam} \quad \text{— (d)}$$

$$f_s = k\,\delta_{spring} = k_{beam} * \delta_{beam} \quad \text{— (e)}$$

Substituting, in eqn (d)

$$\frac{f_s}{k_e} = \frac{f_s}{k} + \frac{f_s}{k_{beam}}$$

$$k_e = \frac{k * k_{beam}}{k + k_{beam}}$$

for simply supported beam,

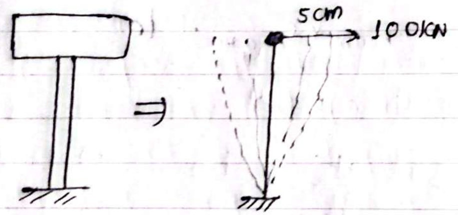
$$k_e = \frac{k * \left(\frac{48EI}{L^3}\right)}{k + \left(\frac{48EI}{L^3}\right)}$$

3. Natural frequency, $\omega_n = \sqrt{\frac{k_e}{m}}$

Q 2.(a) A free vibration test is conducted on an empty elevated water tank. A cable attached to the water tank applies a lateral force of 100kN and pulls the tank horizontally by 5cm. The cable is suddenly cut and the resulting free vibration is recorded. At the end of five complete cycles, the time is 2.0sec, and the amplitude is 2.5cm. From the given data, compute the following

(i) Damping ratio (ii) Natural period of undamped vibration
(iii) Stiffness (iv) Weight
(v) Damping coefficient
(vi) Number of cycles required for the displacement amplitude to decrease to 1cm.

→ Sol^n.



For five cycles, $T_5 = 2.0sec$

Amplitude, $v_5 = 2.5cm$

$F = k * \Delta$

or, $k = \dfrac{100*10^3}{0.05} = 2*10^6 N/m$

For one cycle,

$T_1 = T_D = \dfrac{2}{5} = 0.4sec.$

$v_0 = 5cm$

$v_5 = 2.5cm$

logarithmic decrement, $\delta = \dfrac{1}{m} \ln\left(\dfrac{v_1}{v_{1+m}}\right) = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}}$

$\Rightarrow \dfrac{1}{5} \ln\left(\dfrac{5}{2.5}\right) = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}}$

or, $0.1386 = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}}$

$\Rightarrow [\xi = 0.022]$

(i) Damping ratio $(\xi) = 2.2\%$.

(ii) $\omega_D = \omega\sqrt{1-\xi^2}$

or, $\dfrac{2\pi}{T_D} = \dfrac{2\pi}{T}\sqrt{1-\xi^2}$

or, $T = T_D\sqrt{1-\xi^2}$

Natural period, $T = 0.4\sqrt{1-(0.022)^2} = 0.3999sec.$

(iii) Stiffness, $k = 2*10^6 N/m$

(iv) weight, $\omega = \sqrt{k/m}$

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.3999} = 15.712 \ rad/sec$

$F_c = c \times \dot{v}$

or $m = \frac{k}{\omega^2}$

$= \frac{2 \times 10^6}{(15.712)^2}$

$= 8101.53 kg$

Thus,

weight $(W) = mg = 8101.53 \times 9.81$

$= 79.4761 kN$.

(v) Damping coefficient,

$C = 2m\omega\xi$

$= 2 \times 8101.53 \times 15.712 \times 0.022$

$= 5600.814$ N-s/m.

(vi) $\delta = \frac{1}{m} \ln\left(\frac{v_n}{v_{n+m}}\right)$

or $0.1386 = \frac{1}{m} \ln\left(\frac{5}{1}\right)$

$\Rightarrow [m = 11.61$ cycles$]$

(078BCE198)

Q) 2(b) Determine the dynamic response of the tower (fig. a) subjected to a blast loading as shown in fig. (b). Use Duhamel integral and neglect damping effect. Also, calculate the maximum response.

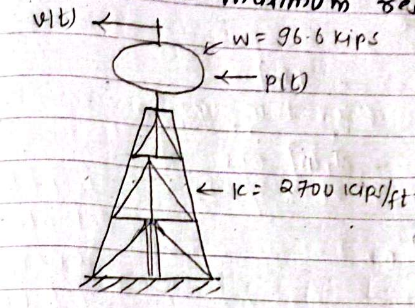


Fig. (a)

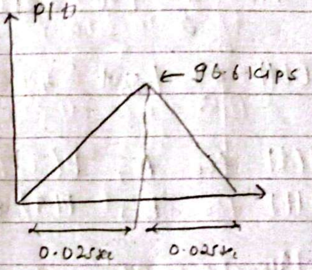


Fig. (b)

→ Solution:-

Weight (wt); $W = 96.6$ kips

Stiffness, $k = 2700$ kips/ft

Blast loading is triangular with

Peak force, $P_0 = 96.6$ kips

$t_1 = 0.025 + 0.025 = 0.05$ sec.

→ mass $(m) = \frac{W}{g} = \frac{96.6 \text{ kips}}{32.2 \text{ ft/sec}^2} = 3 \text{ kips-s}^2/\text{ft}$.

Natural frequency $(\omega_n) = \sqrt{\frac{k}{m}} = \sqrt{\frac{2700}{3}} = 30$ rad/s

Natural period $(T_n) = \frac{2\pi}{\omega_n} = \frac{2\pi}{30} = 0.2094$ sec.

Duhamel Integral,

$$v(t) = \frac{1}{m\omega}\int_0^t P(\tau)\sin\omega(t-\tau)\,d\tau$$

$$= \frac{1}{m\omega}\left[A(t)\sin\omega t - B(t)\cos\omega t\right]$$

where

$$A(t) = \frac{\Delta\tau}{2}\left[J_0 + 2J_1 + 2J_2 + \dots + J_n\right]$$

$$B(t) = \frac{\Delta\tau}{2}\left[J_0 + 2J_1 + 2J_2 + \dots + J_n\right]$$

$J(t)$ for A; $P(\tau)\cos(\omega\tau)$

$J(t)$ for B, $P(\tau)\sin(\omega\tau)$

lets take, $\Delta\tau = 0.01s$, $m\omega = \cancel{3*12*30} = 3\ \text{kips-s}^2/\text{ft} * 30\ \text{rad/s}$

$$= \frac{3*1000}{12}\ \text{lb-s}^2/\text{in} * 30\ \text{rad/s}$$

$$= 7500$$

τ	$\omega\tau$	(kips) $P(\tau)$	$(J(t))_A$ $P(\tau)\cos(\omega\tau)$	$A(t)$	$(J(t))_B$ $P(\tau)\sin(\omega\tau)$	$B(t)$	$v(t)$
0	0	0	0	–	0	–	(Duhamel consider at rest at origin)
0.01	0.3	38.64	36.91	0.184	11.42	0.057	$-1.05\times10^{-5} \approx 0$
0.02	0.6	77.28	63.78	0.688	43.63	0.332	0.0152
0.03	0.9	77.28	48.04	1.247	60.53	0.853	0.0595
0.04	1.2	38.64	14.00	1.557	36.01	1.336	0.129
0.05	1.5	0	0	1.627	0	1.516	0.202

→ Responce during forced vibration phase $(0 \le t \le 0.05\text{sec})$

Using Duhamel's integral for undamped system,

$$v(t) = \frac{1}{m\omega}\int_0^t P(\tau)\sin\omega(t-\tau)\,d\tau.$$

For rising phase $(0 \le t \le 0.025\text{sec})$

$$\therefore P(t) = \begin{cases} \frac{P_0}{t_r} * t, & 0 \le t \le t_r \\ P_0\left(1 - \frac{t-t_r}{t_f}\right), & t_r < t \le t_r + t_f \\ 0, & t > t_r + t_f \end{cases}$$

$t_r = 0.025s$ (rise)

$t_f = 0.025s$ (fall)

$P_0 = 96.6$ kips.

Thus For rising phase,

$$v(t) = \frac{1}{\underset{3\ \ \ 30}{m\omega}}\int_0^{t=0.025} \frac{96.6}{0.025} * \tau \sin[30(0.025-\tau)]\,d\tau.$$

$= 3.26*10^{-3}$ ft

$= 0.039$ in.

For falling phase

$$v(t) = \frac{1}{3*30}\int_{0.025}^{0.025} * 96.6\left(1 - \frac{\tau - 0.025}{0.025}\right)\sin[30(l$$

(078BCE178)

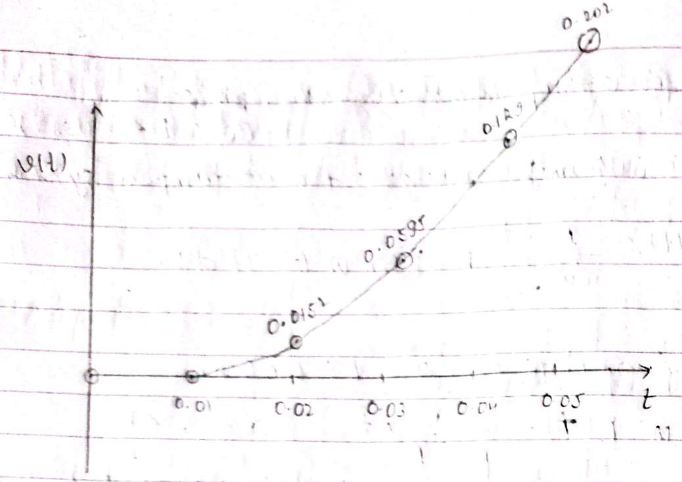


Fig. Response

Max. Response:

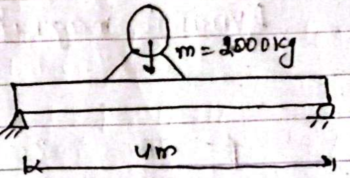
$$v_{max} = \frac{1}{m\omega}\sqrt{A(t)^2 + B(t)^2}$$

$$= \frac{1}{7500}\sqrt{(1.627)^2 + (1.516)^2} \times 1000$$

$= 0.296$ inch

Q.3@ An electric motor of mass 2000kg is mounted on the centre of simple RC beam of a size 230*230 mm² and span of 4m as shown in figure below. The motor runs at a speed of 2000rpm and its rotor is out of balance to the extent of 15kg at a radius of 310mm. Find the amplitude of the steady state response, if equivalent viscous damping for the system, is assumed 5% of critical damping, Neglect mass of beam.

→ Sol:



$m = 2000kg$

$L = 4m$

$\bar{f} = 2000rpm$

$b = 230mm$

$d = 230mm.$

$r = 310mm$

$m' = 15kg$

$\xi = 5\%$

Moment of inertia, $I = \frac{230 \times 230^3}{12} = 233.201 \times 10^6 \, mm^4$

det. RC beam of M20 grade concrete. Then,

$E = 5000\sqrt{f_{ck}} = 5000\sqrt{20}$ (IS456:2

$= 22360.68 \, N/mm^2.$

$EI = 5.214 \times 10^{12} \, Nmm^2$

$= 5214.53 \, KN m^2.$

For simply supported beam,

$\Delta = \frac{PL^3}{48EI} \Rightarrow k = \frac{48EI}{L^3} = \frac{48 \times 5214.53}{4^3}$

$= 3910.9 \, KN/m$

$\omega = \sqrt{K/m} = \sqrt{\dfrac{3910.9 \times 10^3}{2000}} = 44.22$ rad/sec

$\overline{\omega} = 2\pi f = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 2000}{60} = 209.44$ rad/sec.

Amplitude of steady state respone $= \delta$

$\delta = \dfrac{P_0}{k} \times (D)$

Dynamic magnification factor (D)

$$D = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}$$

$\beta = \dfrac{\overline{\omega}}{\omega} = \dfrac{209.44}{44.22} = 4.736$

$P_0 = m' r \overline{\omega}^2$
$= 15 \times 0.310 \times (209.44)^2$
$= 203972.78$ N.

(due to displacement of mass from centre induced force)

$\xi = 5\%$

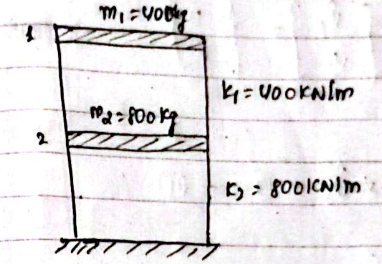
Thus,

$$D = \frac{1}{\sqrt{(1-4.736^2)^2 + (2 \times 0.05 \times 4.736)^2}} = 0.0466$$

Thus,

$$\delta = \frac{203972.8}{3910.9 \times 10^3} \times 0.0466 = 2.43 \text{ mm}$$

Q. 3 (b) Find the natural frequencies and natural vibration mode shapes of the 2-storeyed building with infinitely rigid floors as shown in figure below. Also, draw the mode shapes and write down the modal matrix.



→ Soln:

$[m] = 400 \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ kg

$[k] = 400 \times 10^3 \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$ N/m

Characteristic equation:-

$|[k] - \omega_n^2 [m]| = 0$

or, $\left| 400 \times 10^3 \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix} - \omega_n^2 \times 400 \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \right| = 0$

or, $400 \times 10^3 \begin{vmatrix} (1-P) & -1 \\ -1 & (3-2P) \end{vmatrix} = 0$

where, $P = \dfrac{400\,\omega_n^2}{400 \times 10^3}$

on expanding, $(1-P)(3-2P) - 1 = 0$

$\Rightarrow P_1 = 0.5 \longrightarrow \omega_1 = 22.361$ rad

$P_2 = 2 \longrightarrow \omega_2 = 44.721$ ra

Natural freque

(078BCE179)

For mode shapes,

Eigen value problem is

$$[[K] - \omega_n^2[m]]\{\hat{v}_n\} = \{0\}$$

or
$$\begin{bmatrix} (1-P) & -1 \\ -1 & (3-2P) \end{bmatrix} \begin{Bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

For $\hat{v}_{1n} = 1$,

$$-1 + (3-2P) * \hat{v}_2 = 0 \quad \text{— (11)}$$

For 1st mode, $P_1 = 0.5$

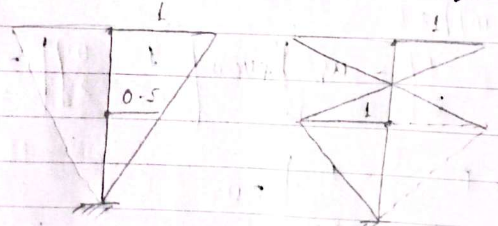
$$\Rightarrow \hat{v}_2 = 0.5$$

For 2nd mode $P_2 = 2$

$$\hat{v}_2 = -1.$$

Thus,

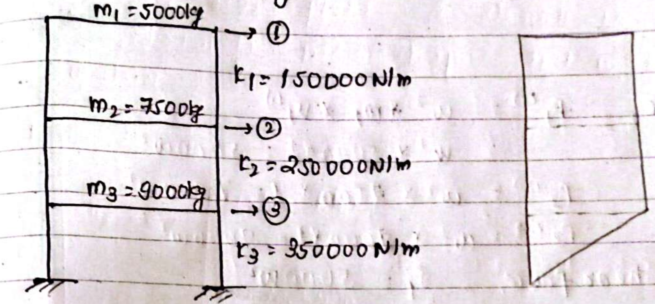
Mode shape matrix, $[\Phi] = \begin{bmatrix} 1 & 1 \\ 0.5 & -1 \end{bmatrix}$



1st mode

2nd mode

(Q) Using improved Rayleigh (R_{00}, R_{01} and R_{11}) method, determine fundamental frequency of vibration of three storey building frame shown in figure below



→ Soln:

Assume shape, $\left.\begin{aligned} \psi_1^{(0)} &= 1.0 \\ \psi_2^{(0)} &= 1.0 \\ \psi_3^{(0)} &= 1.0 \end{aligned}\right\}$ & $Z_0^{(0)}\ \psi_i^{(0)} = 1$

$\Rightarrow Z_0^{(0)} = 1$

$\psi_i^{(0)} = 1$

Thus,

R_{00}:-

$$KE = T_{max}^{(0)} = \frac{1}{2} * \Sigma m_i (\dot{v}_i^{(0)})^2 \qquad \{\dot{v}_i^{(0)} = \omega^2 * Z_0^{(0)} * \psi_i^{(0)}\}$$

$$= \frac{1}{2} * \omega^2 (Z_0^{(0)})^2 * \Sigma m_i * (\psi_i^{(0)})^2$$

$$= \frac{1}{2}\omega^2 * 1 * [5000*1^2 + 7500*1^2 + 9000*1^2]$$

$$[T_{max}^{(0)} = 10750\omega^2] \qquad \{\because \Delta v_i = Z_0^{(0)} * \Delta\psi_i^{(0)}\}$$

$$PE = V_{max}^{(0)} = \frac{1}{2}\Sigma k_i (\Delta v_i)^2 = \frac{1}{2} * (Z_0^{(0)})^2 * \Sigma k_i * (\Delta\psi_i^{(0)})^2$$

$$= \frac{1}{2} * 1^2 * [150000 * 0^2 + 250000 * 0^2 + 350000 * 1^2]$$

$$[V_{max}^{(0)} = 175000].$$

Thus, From energy conservation

or $175000 = 10750\,\omega^2$

$\Rightarrow \boxed{\omega = 4.03 \text{ rad/s}} \leftarrow R_{00}$

R_{01}

Inertia force, $P_1^{(1)} = \omega^2 * m_1 * v_1^{(0)}$

$= \omega^2 * 5000 * 1 = 5000\omega^2$

$P_2^{(1)} = \omega^2 * 7500 * 1 = 7500\omega^2$

$P_3^{(1)} = \omega^2 * 9000 * 1 = 9000\omega^2$

Thus, Shear force, $Q_1 = 5000\omega^2$

$Q_2 = 12500\,\omega^2$

$Q_3 = 21500\,\omega^2.$

& $\Delta v_1 = \dfrac{5000\omega^2}{150000} = \dfrac{\omega^2}{30}$

$\Delta v_2 = \dfrac{12500\omega^2}{250000} = \dfrac{\omega^2}{20}$

$\Delta v_3 = \dfrac{21500\omega^2}{350000} = \dfrac{43\omega^2}{700}$

New deflection

$v_3^{(1)} = 0 + \Delta v_3 = \dfrac{43\omega^2}{700}$

$v_2^{(1)} = v_3^{(1)} + \Delta v_2 = \dfrac{39\omega^2}{350}$

$v_1^{(1)} = v_2^{(1)} + \Delta v_1 = \dfrac{76\omega^2}{525}$

$V_{max}^{(1)} = \dfrac{1}{2} \sum P_i^{(0)} * v_i^{(1)} = \dfrac{1}{2} * \left[5000\omega^2 * \dfrac{76\omega^2}{525} + 7500\omega^2 * \dfrac{39\omega^2}{350} + 9000\omega^2 * \dfrac{43\omega^2}{700}\right]$

$= 1056.19\,\omega^4.$

Thus, $V_{max}^{(1)} = T_{max}^{(0)}$

or $1056.19\omega^4 = 10750\omega^2$

$\Rightarrow \boxed{\omega = 3.19 \text{ rad/s}} \leftarrow R_{01}$

R_{11}

$v_i^{(1)} = \omega^2\, \varphi_i^{(1)}\, \bar{Z}_0^{(1)}$

Here,

$v_1^{(1)} = \dfrac{76\omega^2}{525} = \bar{Z}_0 * \varphi_1^{(1)} * \omega^2$

Assuming, top floor shape as 1.0

i.e. $\varphi_1^{(1)} = 1$

Normalizing other] Then, $\varphi_2^{(1)} = \dfrac{39/350}{76/525} = 0.77$

$\varphi_3^{(1)} = \dfrac{43/700}{76/525} = 0.424.$

$T_{max}^{(1)} = \dfrac{\omega^6}{2} [\bar{Z}_0^{(1)}]^2 \sum m_i [\varphi_i^{(1)}]^2$

$= \dfrac{\omega^6}{2} * \left[\dfrac{76}{525}\right]^2 * [5000 * 1^2 + 7500 * 0.77^2 + 9000 * 0.4$

$= 115.936\,\omega^6$

Thus, $V_{max}^{(1)} = T_{max}^{(1)}$

or $1056.19\omega^4 = 115.936\omega^6$

$\Rightarrow \boxed{\omega = 3.02 \text{ rad/s}} \leftarrow R_{11}$

(078BCE179)

Q.5

(c) Method of Discretization

(1) Lumped Mass Method (LMM)

→ Assume that the mass of the structure is concentrated (lumped) at certain discrete points (usually floor levels in buildings).

→ Inertia forces develop only at those mass points.

→ DOF equal to the number of mass points * possible directions of motion.

→ Simple, effective when mass is really concentrated at few points.

→ Eg. Treating a 3-storey building as a 3-DOF shear frame with floor masses lumped.

(2) Generalized coordinates method (GCM)

→ Represent the displacement shape of the structure as a combination of assumed functions/shape functions, sine/cosine waves, polynomials.

→ Converts infinite DOF into a finite number by truncating the series.

→ Better approximation than LMM for distributed-mass structures, but requires more computation.

→ Eg. Beam deflection represented as a series of sine functions:

$$v(x) \approx b_1 \sin\frac{\pi x}{L} + b_2 \sin\frac{2\pi x}{L} + \ldots$$

(3) Finite Element Method (FEM)

→ ~~Represents the displacements~~

→ Divide the structure into small elements connected at nodes. Nodal displacements are taken as generalized coordinates.

→ Interpolation function (shape function) describe deformation inside each element.

→ Adv:

↳ Works for 1D (beams), 2D (plates, shells) and 3D solids

↳ Efficient for computer analysis

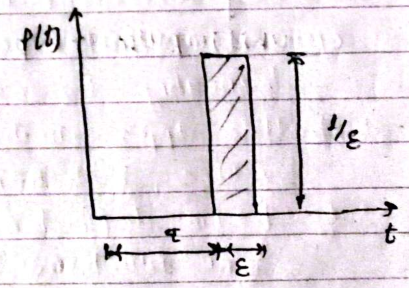
↳ Handles complex geometry, boundary conditions and loading.

→ Eg: Beam divided into finite elements with nodal deflections and rotations as DOF.

→ Most versatile; can achieve high accuracy

(d) Duhamel integral and convolution integral

→ Response due to unit impulse



→ $I = p(t) * dt$

$I = m * \Delta\dot{v}$

or $\Delta\dot{v} = \frac{I}{m}$

For unit impulse,

$\Delta\dot{v} = 1/m$

The unit impulse at $t=\tau$ imparts the velocity given by:-

$$\dot{v}(\tau) = \frac{1}{m}$$

but displacement prior to impulse

$$v(\tau)=0$$

A unit impulse causes free vibration of the SDOF system due to initial velocity and displacement at $t=\tau$. Hence response due to unit impulse

• For undamped free vibration:

$$v(\bar{t}) = v(\tau)\cos\omega\bar{t} + \frac{\dot{v}(\tau)}{\omega}\sin\omega\bar{t}$$

$$\{\bar{t} = t-\tau\}$$

or $$h(t-\tau) \equiv v(\bar{t}) = v(\tau)\cos\omega(t-\tau) + \frac{\dot{v}(\tau)}{\omega}\sin\omega(t-\tau)$$

On substitution

$$\left[\underbrace{h(t-\tau)}_{\text{unit impulse response function}} \equiv v(\bar{t}) = \frac{1}{m\omega}\sin\omega(t-\tau)\right] \text{— (I)}; \; t \geq \tau$$

• For damped free vibration:

$$h(t-\tau) \equiv v(\bar{t}) = e^{-\xi\omega\bar{t}}\left[v(\tau)\cos\omega_D\bar{t} + \frac{\dot{v}(\tau)+v(\tau)\omega}{\omega_D}\sin\omega_D\bar{t}\right]$$

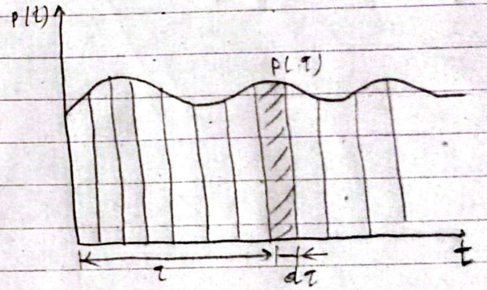
where, $\omega_D = \omega\sqrt{1-\xi^2}$

On substitution

On $$\left[h(t-\tau) \equiv v(\bar{t}) = e^{-\xi\omega(t-\tau)} * \frac{1}{m\omega_D}\sin\omega_D(t-\tau)\right]; \; t \geq \tau \text{ — (II)}$$

↳ Response to Arbitrary Force / General dynamic loading

Response to linear dynamic system for one of these impulses, the one at time τ of magnitude $I = P(\tau)\,d\tau$ is given by:



$$dv(t) = I * h(t-\tau)$$

(sequence of infinitesimally short impulses)

or, $$dv(t) = P(\tau)d\tau * h(t-\tau) \; ; t \geq \tau$$

Total response of system at time 't' is sum of all impulses response upto that time 't':

$$\left[v(t) = \int_0^t dv(t) = \int_0^t P(\tau)\cdot h(t-\tau)\,d\tau\right] \text{ — (III)}$$

↳ Convolution Integral / Superposition integral.

Substitution of $h(t-\tau)$ in convolution integral gives Duhamel integral,

- For undamped system (from (I) & (III))

$$\left[v(t) = \frac{1}{m\omega}\int_0^t p(\tau)\sin\{\omega(t-\tau)\}d\tau\right]$$

- For damped system (from (II) & (III))

$$\left[v(t) = \frac{1}{m\omega_D}\int_0^t p(\tau)\, e^{-\xi\omega(t-\tau)}\sin\{\omega_D(t-\tau)\}d\tau\right]$$